Lectures on Economic Inequality

Warwick, Summer 2018, Slides 4

Debraj Ray

- Inequality and Divergence I. Personal Inequalities, Slides 1 and 2
- Inequality and Divergence II. Functional Inequalities, Slides 3
- Inequality and Conflict I. Polarization and Fractionalization, Slides 4
- Inequality and Conflict II. Some Empirical Findings
- Inequality and Conflict III. Towards a Theory of Class Conflict

Slides 4. Inequality and Conflict: Polarization and Fractionalization

- Uneven Growth:
- **Roots**
- Technological change
- Structural transformation
- Globalization

Slides 4. Inequality and Conflict: Polarization and Fractionalization

- Uneven Growth:
- **Roots**
- Technological change: (leading to jumps in inequality, only slowly eroded)
- Structural transformation
- Globalization

Slides 4. Inequality and Conflict: Polarization and Fractionalization

- Uneven Growth:
- **Roots**

- Technological change
- Structural transformation: (agriculture $\rightarrow$ industry, manufacturing $\rightarrow$ services)
- Globalization

Slides 4. Inequality and Conflict: Polarization and Fractionalization

- Uneven Growth:
- **Roots**
  - Technological change
  - Structural transformation
  - Globalization: (sectors with comparative advantage)

Slides 4. Inequality and Conflict: Polarization and Fractionalization

- Uneven Growth:
- **Roots**
  - Technological change
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- **Reactions**
  - Occupational choice
  - Cross-sector percolation
  - Political economy
  - Conflict

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- Uneven Growth:
- **Roots**
  - Technological change
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- **Reactions**
  - Occupational choice: (slow, imprecise, intergenerational)
  - Cross-sector percolation
  - Political economy
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- Uneven Growth:
- **Roots**
  - Technological change
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  - Occupational choice
  - Cross-sector percolation: (demand patterns, inflation)
  - Political economy
  - Conflict

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- Uneven Growth:
- **Roots**
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  - Occupational choice
  - Cross-sector percolation
  - Political economy: (lobbying for tax breaks, favored allocations)
  - Conflict

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- Uneven Growth:
- **Roots**
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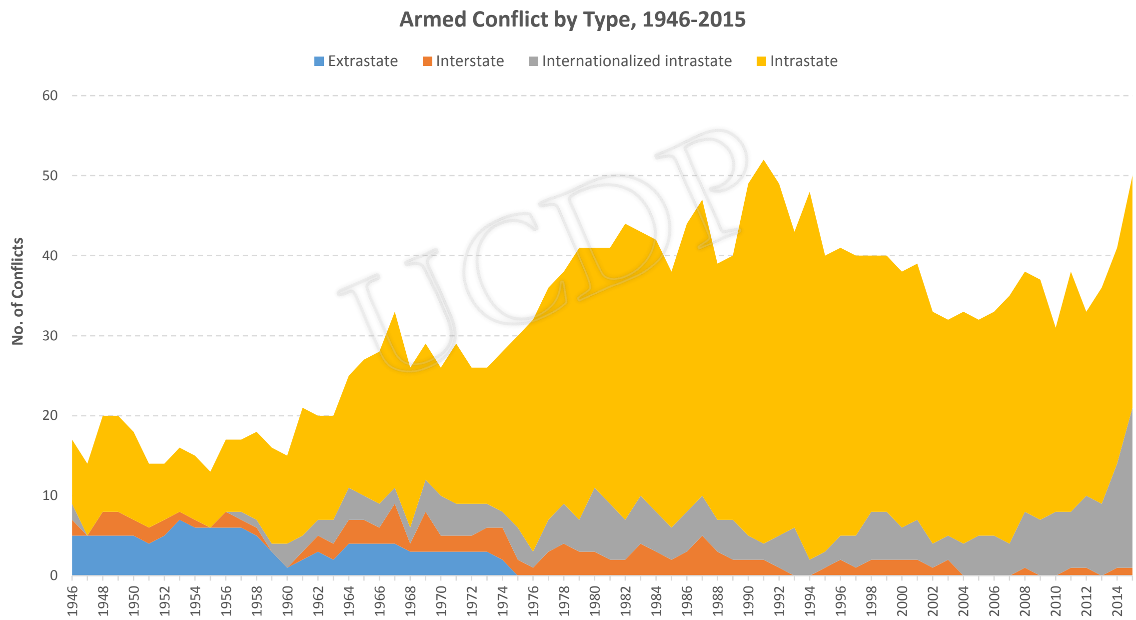
  - Occupational choice
  - Cross-sector percolation
  - Political economy
  - Conflict (this lecture)

The Backlash Against Uneven Growth

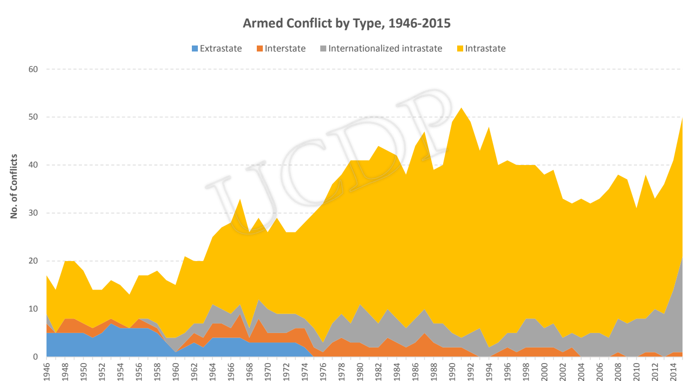
- The lives of others on display
  - (on an accelerating treadmill)
- Aspirations and frustrations are socially generated.
  - Unclear if this exposure leads to betterment or to despair.

Hirschman's Tunnel

Internal Social Conflict



Internal Social Conflict



- WWII → 2000: 240 *intrastate* armed conflicts:
  - Battle deaths 5–10m (3–8 m for interstate)
  - Mass assassination (25m civilians), forced displacement (60m civilians)
  - In 2015: 29 ongoing conflicts.

UCDP/PRIO definition: armed conflict, 25+ yearly deaths.

Majority of Internal Conflicts are Ethnic

- 1945–1998, 100 of 700 known ethnic groups participated in rebellion against the state. Fearon 2006

- "[T]he eclipse of the left-right ideological axis." Brubaker and Laitin (1998)

- "In much of Asia and Africa, it is only modest hyperbole to assert that the Marxian prophecy has had an ethnic fulfillment." Horowitz 1985

Ethnicity or Class?

- One of the great questions of political economy:
  - It isn't that the Marxian view is entirely irrelevant, but . . .
- Economic similarity appears to matter as much or even more.
  - Conflict is usually over *directly contested resources*:
  - land, jobs, business resources, government quotas . . .
- The implications of direct contestation:
  - Ethnic markers.
  - Instrumentalism as opposed to primordialism (Huntington, Lewis)

Do Ethnic Divisions Matter?

- Two ways to approach this question.
- Historical study of conflicts, one by one.
  - Bit of a wood-for-the-trees problem.
  - Horowitz (1985) summarizes some of the complexity:

"In dispersed systems, group loyalties are parochial, and ethnic conflict is localized . . . A centrally focused system [with few groupings] possesses fewer cleavages than a dispersed system, but those it possesses run through the whole society and are of greater magnitude. When conflict occurs, the center has little latitude to placate some groups without antagonizing others."

- Statistical approach
  - (Collier-Hoeffler, Fearon-Laitin, Miguel-Satyanath-Sergenti)
- Typical variables for conflict: demonstrations, processions, strikes, riots, casualties and on to civil war.
- Explanatory variables:
  - Economic. per-capita income, inequality, resource holdings . . .
  - Geographic. mountains, separation from capital city . . .
  - Political. "democracy", prior war . . .
  - And, of course, Ethnic. But how measured?

- Information on ethnolinguistic diversity from:
  - World Christian Encyclopedia
  - Encyclopedia Britannica
  - Atlas Narodov Mira
  - CIA FactBook
- Or religious diversity from:
  - L'Etat des Religions dans le Monde
  - World Christian Encyclopedia
  - The Statesman's Yearbook

Fractionalization

- Fractionalization index widely used:

$$F = \sum_{j=1}^{m} n_j(1 - n_j)$$

where n_j is population share of group j.

- Special case of the Gini coefficient

$$G = \sum_{j=1}^{m} \sum_{k=1}^{M} n_j n_k \delta_{ik}$$

where δ_{ik} is a notion of distance across groups.

- Fractionalization used in many different contexts:
  - growth, governance, public goods provision.
  - But it shows no correlation with conflict.
  - Collier-Hoeffler (2002), Fearon-Laitin (2003), Miguel-Satyanath-Sergenti (2004)
  - Fearon and Laitin (*APSR* 2003):

"The estimates for the effect of ethnic and religious fractionalization are substantively and statistically insignificant . . . The empirical pattern is thus inconsistent with . . . the common expectation that ethnic diversity is a major and direct cause of civil violence."

- And yet . . . fractionalization does not seem to capture the Horowitz quote.
- Motivates the use of polarization measures.

The Identity-Alienation Framework

- Society is divided into "groups" (economic, social, religious, spatial...)
- Identity. There is "homogeneity" within each group.
- Alienation. There is "heterogeneity" across groups.
- Esteban and Ray (1994) presumed that such a situation is conflictual:

"We begin with the obvious question: why are we interested in polarization? It is our contention that the phenomenon of polarization is closely linked to the generation of tensions, to the possibilities of articulated rebellion and revolt, and to the existence of social unrest in general . . . "

Measuring Polarization

- Space of densities (cdfs) on income, political opinion, etc.

- Each individual located at "income" x feels

  - Identification with people of "similar" income (the height of density $n(x)$ at point x.)

  - Alienation from people with "dissimilar" income (the income distance $|y-x|$ of y from x.)

- Effective Antagonism of x towards y depends on x's alienation from y *and* on x's sense of identification.

$$T(i,a)$$

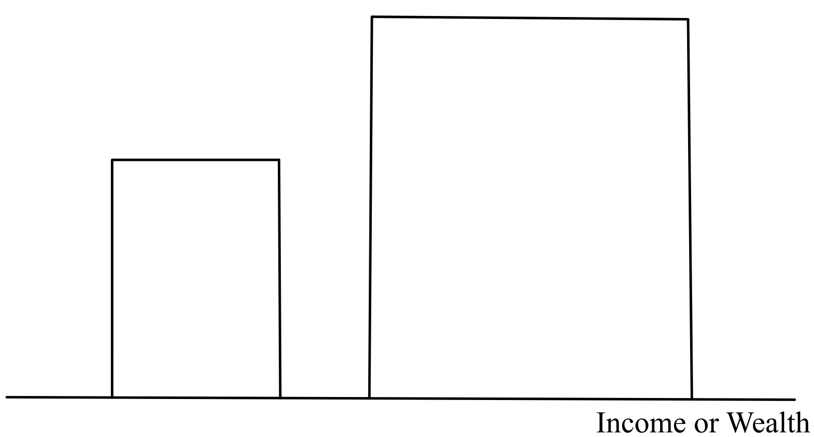
where $i = n(x)$ and $a = |x-y|$.

- View polarization as the "sum" of all such antagonisms

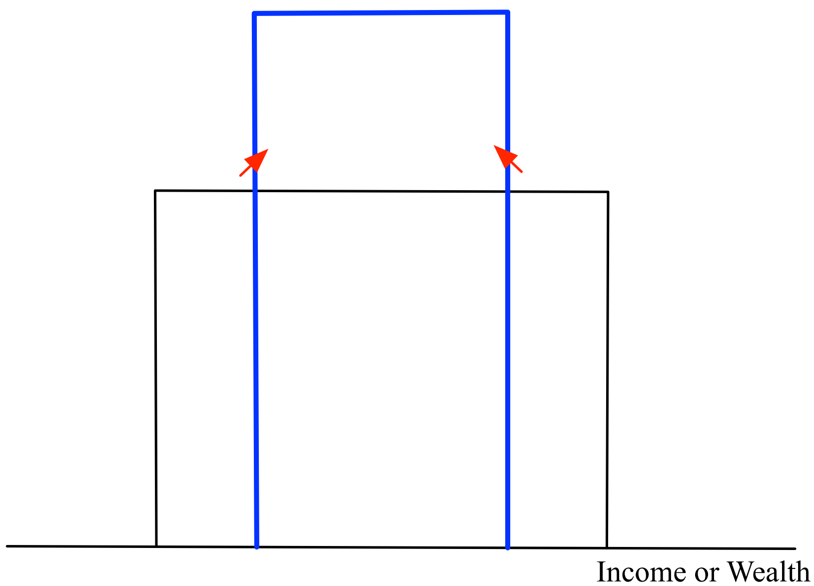
$$P(f) = \int \int T\left(n(x), |x-y|\right) n(x)n(y)dxdy$$

- Not very useful as it stands. Axioms to narrow down P.

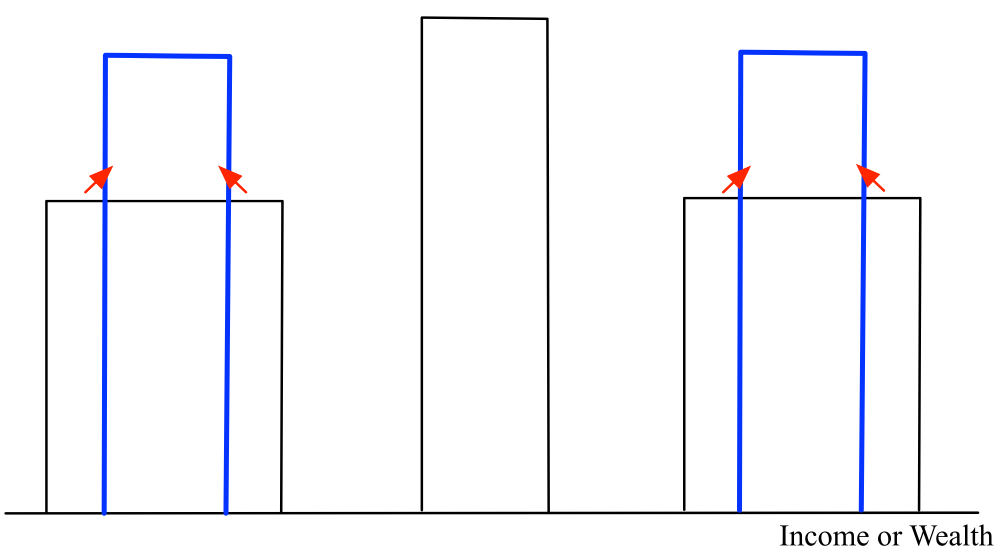
  - Based on special distributions, built from uniform kernels.



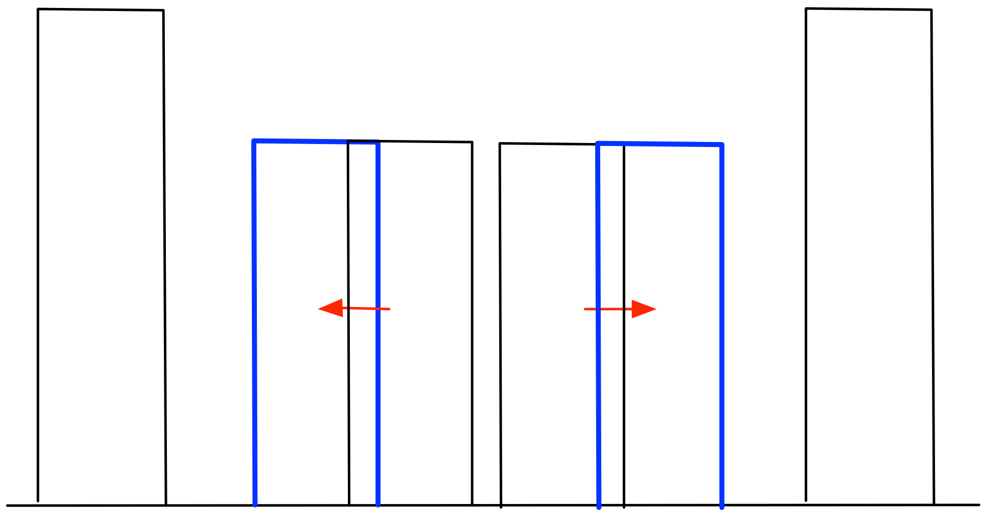
- Axiom 1. If a distribution is just a single uniform density, a “global compression” cannot increase polarization.



- Axiom 2. If a symmetric distribution is composed of three uniform kernels, then a compression of the side kernels cannot reduce polarization.



■ Axiom 3. If a symmetric distribution is composed of four uniform kernels, then a symmetric slide of the two middle kernels away from each other must increase polarization.



■ Axiom 4. [Population Neutrality.] Polarization comparisons are unchanged if both populations are scaled up or down by the same percentage.

■ Theorem. A polarization measure satisfies Axioms 1–4 if and only if it is proportional to

$$\int\int n(x)^{1+\alpha} n(y)|y-x|dydx,$$

where α lies between 0.25 and 1.

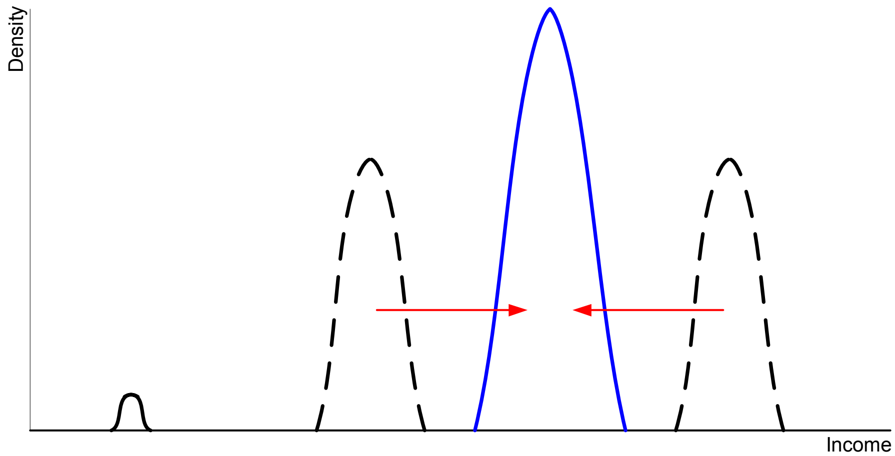
- Compare with the Gini coefficient / fractionalization index:

$$\text{Gini} = \int\int n(x)n(y)|y-x|dydx,$$

- It's α that makes all the difference.

Some Properties

- 1. Not Inequality. See Axiom 2.
- 2. Bimodal. Polarization maximal for bimodal distributions, but defined of course over all distributions.
- 3. Contextual. Same movement can have different implications.



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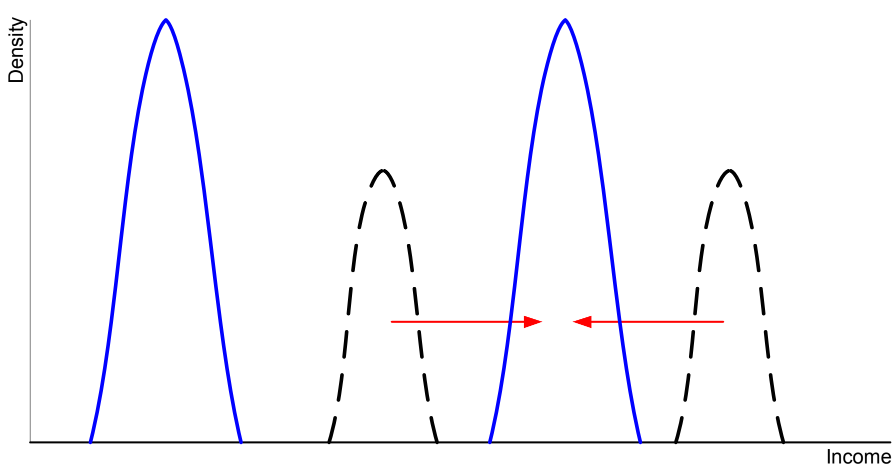
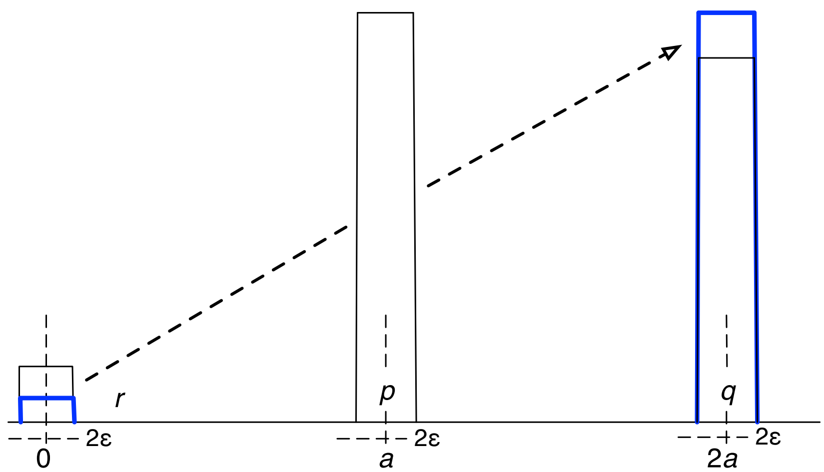


More on α

$$\text{Pol} = \int \int n(x)^{1+\alpha} n(y)|y-x| dydx,$$

where α lies between 0.25 and 1.

- **Axiom 5.** If $p > q$ but $p - q$ is small and so is r, a small shift of mass from r to q cannot reduce polarization.



- **Theorem.** Under the additional Axiom 5, it must be that $\alpha = 1$, so the unique polarization measure that satisfies the five axioms is proportional to

$$\int \int n(x)^2 n(y)|y-x| dydx.$$

- Easily applicable to ethnolinguistic or religious groupings.
  - Say m "social groups", n_j is population proportion in group j.
- If all inter-group distances are binary, then

$$\text{Pol} = \sum_{j=1}^{M} \sum_{k=1}^{M} n_j^2 n_k = \sum_{j=1}^{M} n_j^2 (1 - n_j).$$

  - Compare with $F = \sum_{j=1}^{M} n_j (1 - n_j)$.

Polarization and Conflict: Behavior

- Axiomatics suggest (but cannot establish) a link between polarization and conflict.

- Two approaches:

  - Theoretical. Write down a “natural” theory which links conflict with these measures.

  - Empirical. Take the measures to the data and see they are related to conflict.

We discuss the theory first (based on Esteban and Ray, 2011).

A Theory that Informs an Empirical Specification

- m groups engaged in conflict.

  - n_i: population share of group i, $\sum_{i=1}^{m} n_i = 1$.

- Public prize: payoff matrix $[\ \pi u_{ij}\ ]$ scaled by per-capita size π.

  - (religious dominance, political control, hatreds, public goods)

- Private prize μ per-capita budget, so μ/n_i if captured by group i.

  - Oil, diamonds, scarce land

Theory, contd.

- Individual resource contribution r at convex utility cost $c(r)$.
  - (more generally $c(r, y_i)$).
- R_i is total contributions by group i. Define

$$R = \sum_{i=1}^{m} R_i.$$

- Probability of success given by

$$p_j = \frac{R_j}{R}$$

- R our measure of overall conflict.

Payoffs (per-capita)

- $\pi u_{ii} + \mu / n_i$

(in case i wins the conflict), and

- πu_{ij}

(in case j wins).

- Net per-capita payoff to group i is

$$\Psi_i = \underbrace{\sum_{j=1}^{m} p_j \pi u_{ij}}_{\textbf{pub}} + \underbrace{p_i \frac{\mu}{n_i}}_{\textbf{priv}} - \underbrace{c(r_i)}_{\textbf{cost}}.$$

Contributing to Conflict

- Assume group leader chooses r_i to maximize group per-capita payoff:

$$\Psi_i = \sum_{j=1}^{m} p_j \pi u_{ij} + p_i \frac{\mu}{n_i} - c(r_i).$$

  - Alternative: individuals max combination of own and group payoff.

- Equilibrium: Every group leader unilaterally maximizes group payoffs.

- Theorem 1. An equilibrium exists. If $c'''(r) \geq 0$, it is unique.

- Payoff function for group i:

$$\Psi_i = \sum_{j=1}^{m} p_j \pi u_{ij} + p_i \frac{\mu}{n_i} - c(r_i).$$

- Payoff function for group i:

$$\Psi_i = \sum_{j=1}^{m} p_j \lambda u_{ij} + p_i \frac{(1-\lambda)}{n_i} - \frac{1}{\pi+\mu} c(r_i).$$

- Payoff function for group i:

$$\Psi_i = \sum_{j=1}^{m} p_j v_{ij} - \frac{1}{\pi+\mu} c(r_i).$$

where $v_{ii} = \lambda u_{ii} + (1-\lambda)(1/n_i)$ and $v_{ij} = \lambda u_{ij}$ if $j \neq i$.

- First-order conditions:

$$\left[\frac{n_i}{R} v_{ii} - n_i \sum_j \frac{n_j r_j}{R^2} v_{ij}\right] = \frac{1}{\pi+\mu} c'(r_i)$$

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where $\Delta_{ij} = v_{ii} - v_{ij}$.

- Define $\gamma_i = p_i/n_i$. Then

$$\sum_j \gamma_i \gamma_j n_i n_j \Delta_{ij} = \frac{1}{\pi+\mu} r_i c'(r_i)$$

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$$\sum_j \phi(\gamma_i, \gamma_j, R) n_i^2 n_j \Delta_{ij} = \frac{R}{\pi+\mu} p_i c'(R)$$

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- Define $\gamma_i = p_i/n_i$. Then

$$\sum_i \sum_j n_i^2 n_j \Delta_{ij} \simeq \frac{Rc'(R)}{\pi + \mu}$$

- Approximation.

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$$\sum_i \sum_j n_i^2 n_j \lambda \delta_{ij} + \sum_i \sum_{j \neq i} n_i^2 n_j \frac{1-\lambda}{n_i} \simeq \frac{Rc'(R)}{\pi + \mu}$$

- Opening up Δ_{ij} and defining $\delta_{ij} = u_{ii} - u_{ij}$.

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where $v_{ii} = \lambda u_{ii} + (1-\lambda)(1/n_i)$ and $v_{ij} = \lambda u_{ij}$ if $j \neq i$.

- First-order conditions:

$$\sum_j p_i p_j \Delta_{ij} = \frac{1}{\pi+\mu} r_i c'(r_i)$$

where $\Delta_{ij} = v_{ii} - v_{ij}$.

  - Define $\gamma_i = p_i/n_i$. Then

$$\lambda \sum_i \sum_j n_i^2 n_j \delta_{ij} + (1-\lambda) \sum_i n_i (1-n_i) \simeq \frac{Rc'(R)}{\pi+\mu}$$

  - Opening up Δ_{ij} and defining $\delta_{ij} = u_{ii} - u_{ij}$.

Approximation Theorem

- Theorem 2. R "approximately" solves

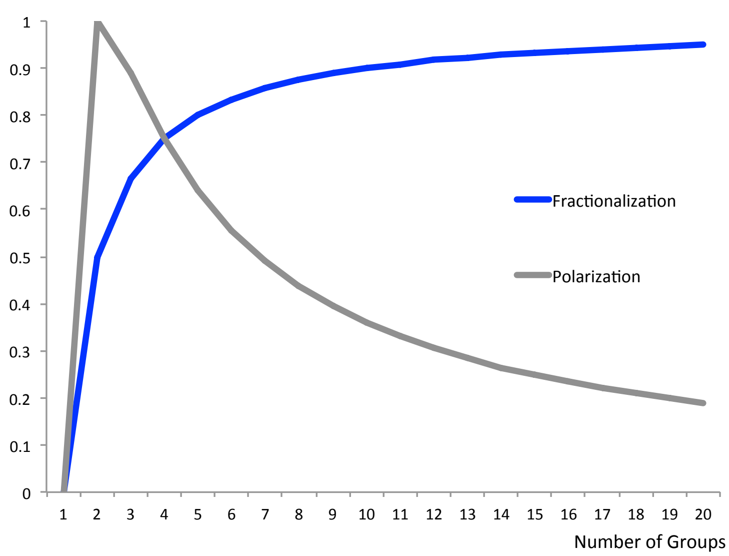
$$\frac{Rc'(R)}{\pi+\mu} = \lambda P + (1-\lambda)F,$$

where

- $\lambda \equiv \pi/(\pi+\mu)$ is relative publicness of the prize.
- P is squared polarization: $\sum_i \sum_j n_i^2 n_j d_{ij}$
- F is fractionalization: $\sum_i n_i(1-n_i)$.
- Note: Theorem is more complex with individual contributions.

Polarization and Fractionalization

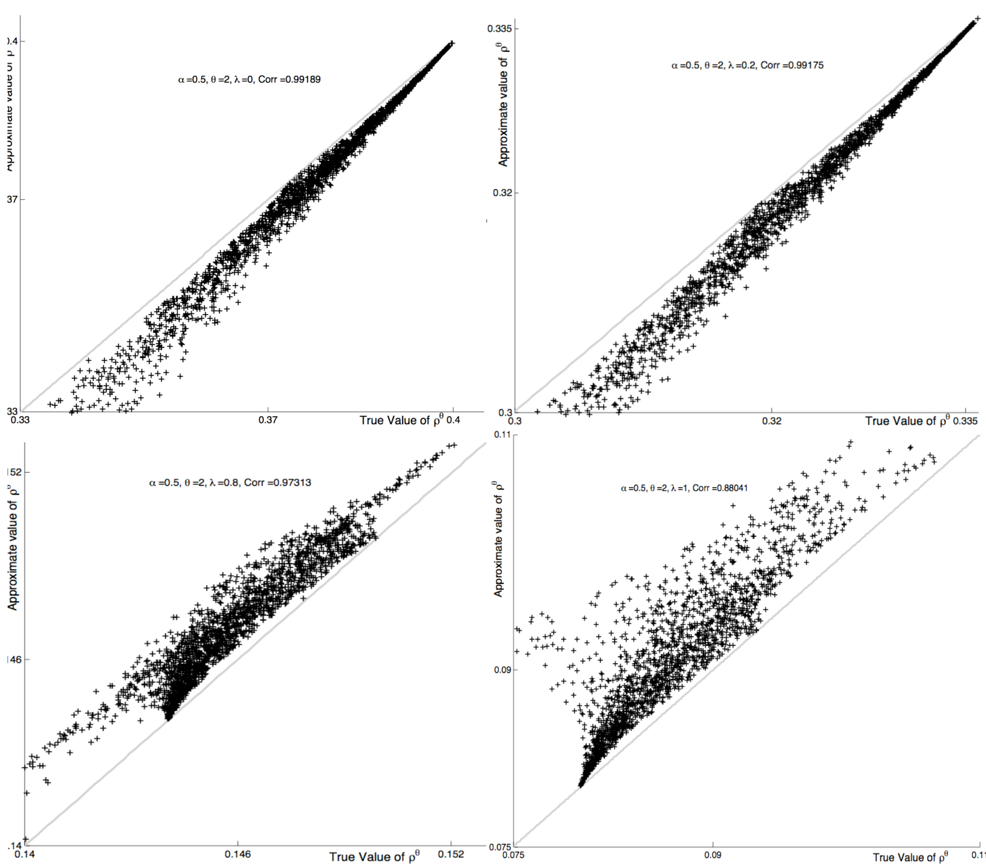
- With $n_i = 1/m$, P maxed at $m = 2$, F increases in m:



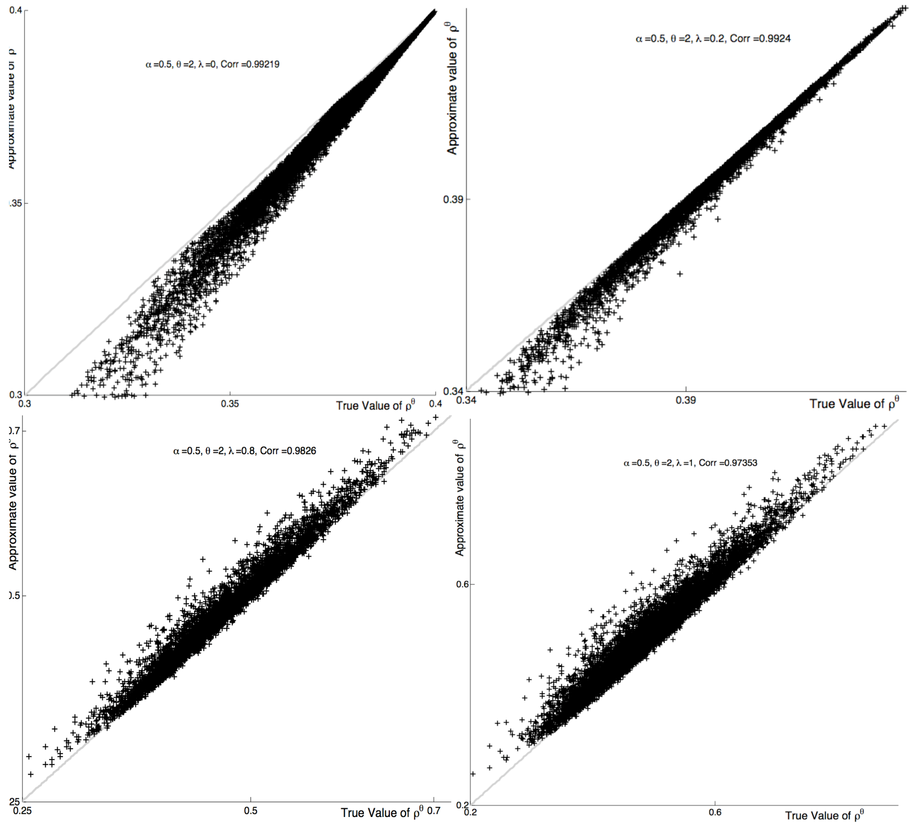
How Good is the Approximation?

- Holds exactly when there are just two groups and all goods are public.
- Holds exactly when all groups the same size and public goods losses are symmetric.
- Holds almost exactly for contests when conflict is high enough.
- Can numerically simulate to see how good the approximation is.

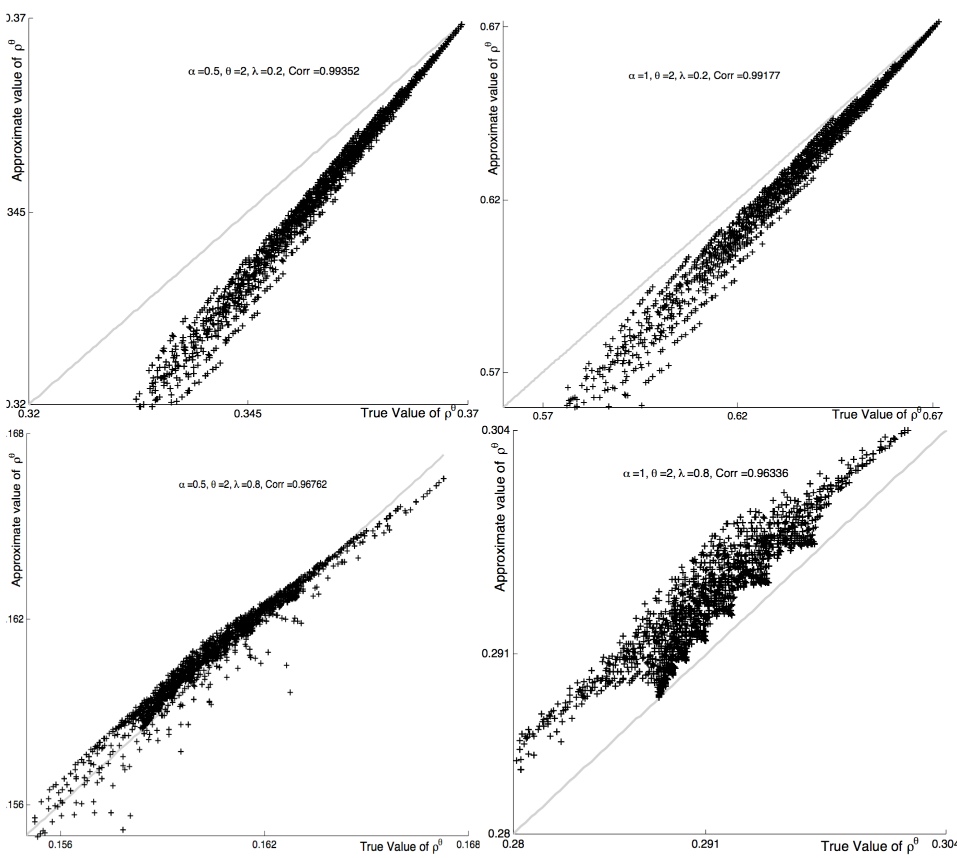
Contests, quadratic costs, large populations, λ various:



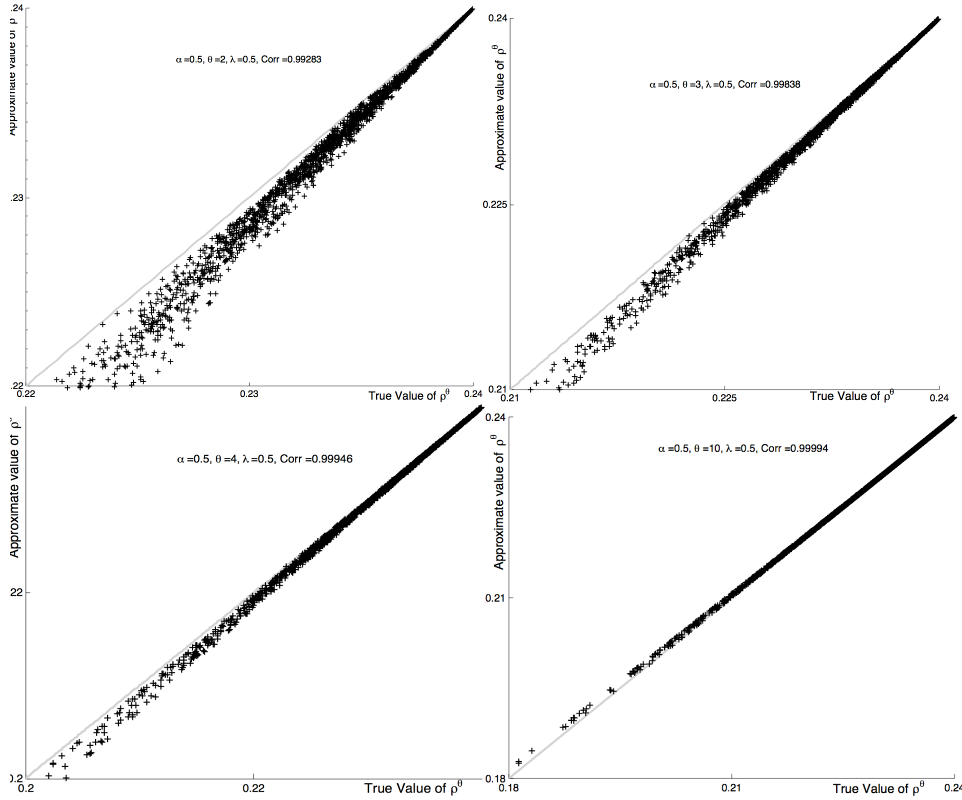
Distances, quadratic costs, large populations, λ various:



Small populations, λ various:



Nonquadratic costs, large populations, λ various:



Summary

- Ethnic conflicts are widespread.

  - Yet studies that relate conflict to ethnic divisions (as measured by fractionalization) show little or no correlation.

- In this lecture we approach the problem from a conceptual perspective:

  - We axiomatize a measure of polarization

  - We argue it is different from fractionalization

  - We argue that *both* polarization and fractionalization should enter the conflict equation.

- We argue that the polarization-conflict nexus is strong when the prize is private, and the fractionalization-conflict nexus is strong when the prize is private.

  - For another parallel exercise with group size, see Supplement to these slides.