Lectures on Economic Inequality

Warwick, Summer 2018, Slides 3

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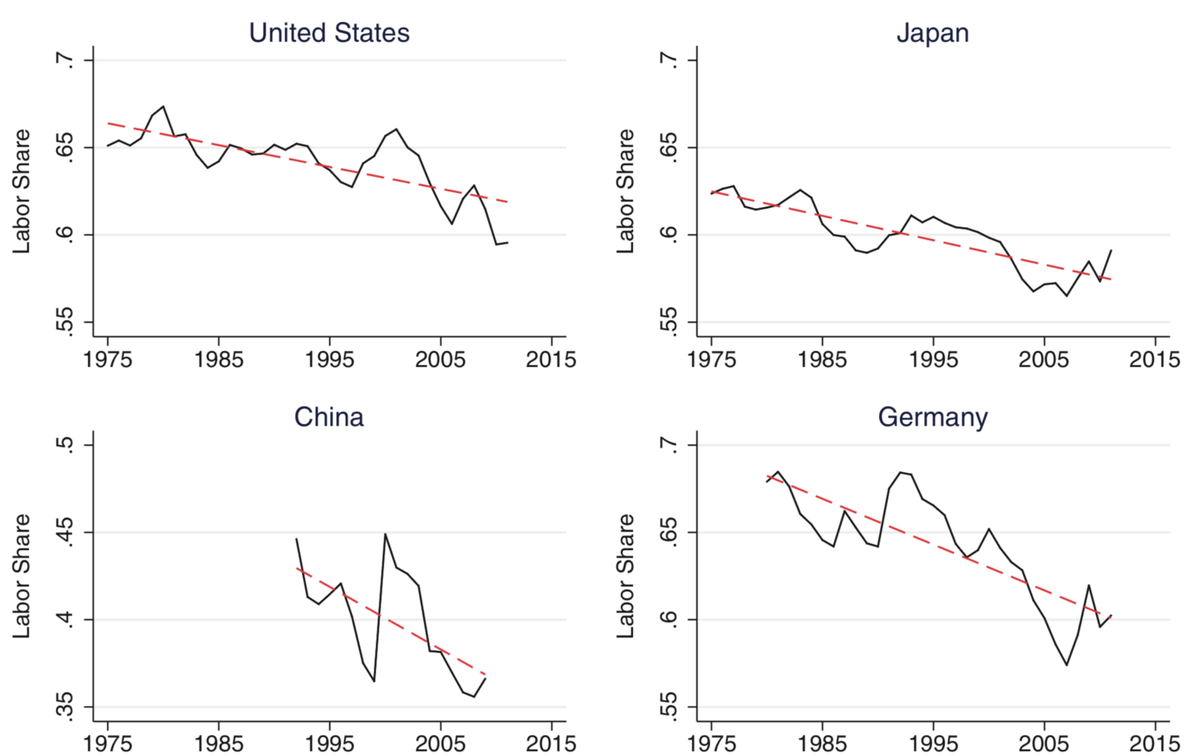
- Inequality and Divergence I. Personal Inequalities, Slides 1 and 2
- Inequality and Divergence II. Functional Inequalities, Slides 3
- Inequality and Conflict I. Polarization and Fractionalization
- Inequality and Conflict II. Some Empirical Findings
- Inequality and Conflict III. Towards a Theory of Class Conflict

Slides 3. Markets and Functional Inequality

- An age-old anxiety: that capital will inherit the earth:
- share of capital + rent income $\uparrow$.
- Statement about functional, not personal distribution.
- Still worrisome.
- Jobless growth is a particularly widespread fear. Reactions:
- universal basic income or share
- universal participation in a stock portfolio

Significant empirical support for a falling labor share; Harrison (2002), Rodríguez and Jayadev (2010), Karabarbounis and Neiman (2014)...

Karabarbounis and Neiman (2014):



Must Capital Persistently Displace Labor?

- Two arguments that are relevant to this "law":

I. Technical Progress

- robotics (hardware), machine learning (software)

- Large literature on directed technical change: Hicks (1932), Drandakis-Phelps (1965), Kennedy (1964), Salter (1966), Autor-Krueger-Katz (1998), Galor-Maov (2000), Acemoglu (1998, 2002), Acemoglu and Restrepo 2017 . . .

- Pulls this way or that depending on which input is relatively scarce. Focus is on balanced growth.

Already There, Though

- But in large part, the technology already exists.

- It's just a question of building the right machines.

"Machines that can learn mean nothing humans do as a job is uniquely safe anymore. From hamburgers to healthcare, machines can be created to successfully perform such tasks with no need or less need for humans, and at lower costs than humans. . .

What's the big lesson to learn, in a century when machines can learn? Maybe it is that jobs are for machines, and life is for people."

Scott Santens, *The Boston Globe*, 2016.

Google's AlphaZero Destroys Stockfish In 100-Game Match

FM MikeKlein
Dec 6, 2017, 12:50 PM | 343 | Chess Event Coverage

English

Chess changed forever today. And maybe the rest of the world did, too.

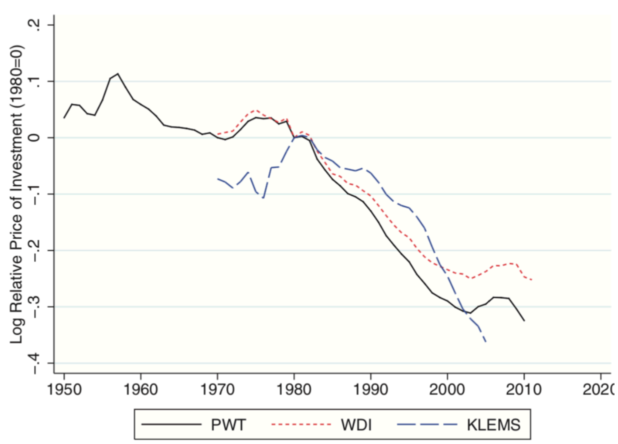
A little more than a year after **AlphaGo** sensationally won against the top Go player, the artificial-intelligence program **AlphaZero** has obliterated the highest-rated chess engine.

Stockfish, which for most top players is their go-to preparation tool, and which won the **2016 TCEC Championship** and the **2017 Chess.com Computer Chess Championship**, didn't stand a chance. AlphaZero won the closed-door, 100-game match with 28 wins, 72 draws, and zero losses.

Oh, and it took AlphaZero only four hours to "learn" chess. Sorry humans, you had a good run.

II. Capital Abundance and its Falling Relative Price

- Parallel to Ricardian theory of land:
  - Labor fixed, capital accumulates, so the relative price of capital falls.
- Induces techniques that replace labor, without technical progress.



- Year fixed effects from regressions of the log relative price of investment that absorb country fixed effects (Karabarbounis and Neiman 2014):

The Elasticity of Substitution

- Capital displaces labor, but the effect on income *shares* depends on σ.

- Here the literature gets more muddy.

- Karabarbounis and Neiman (2014) estimate $\sigma = 1.25$ using cross-country variations in shares and factor prices, and conclude: 'Roughly half of the global decline in the labor share is explained by the decline in the relative price of investment."

- Chirinko and Mallik (2014) estimate $\sigma = 0.4$ from US industry data, and conclude: "[T]he secular decline in the labor share of income cannot be explained by secular increases in the capital/income ratio or capital-augmenting technical change or secular decreases in the relative price of investment or capital taxation."

- See also Autor, Dorn, Kartz, Petterson, Van Reenen 2017

Automation

- A discrete act at the firm or high-digit industry level, where labor is replaced.

- Needs a different model to inform estimation

- Not fixed CES formulation by industry or country

- Perfectly compatible with low elasticity of substitution across existing inputs.

An Equilibrium Model with Robots

- Final goods $i \in [0,1]$, produced by

$$y_i = f_i(k_i, \ell_i),$$

with f_i smooth, CRS, strictly quasi-concave, continuous in index i.

- k is capital, ℓ is human or robot labor.
- That is, $\ell_i = h_i + [r_i/\lambda_i]$, where
- h_i = human input, r_i = robot input, and
- λ_i: sector-specific replacement threshold, continuous increasing in i
- λ_i high implies i is protected for humans
- $0 < \lambda_i < \infty$: no sector can be freely automated or is fully protected.

Two More Sectors

- The robot sector:

$$y_r = f_r(k_r, \ell_r)$$

where $\ell_r = h_r + [r_r/\lambda_r]$, and $0 < \lambda_r < \infty$.

- The education sector:

$$y_e = f_e(k_e, \ell_e)$$

where $\ell_e = h_e + [r_e/\lambda_e]$, and $0 < \lambda_e < \infty$.

- Summary of sectors: $[0,1] \cup r \cup e$, typical index j.

Capital and Labor

- (Non-Robot) Capital $\bar{K}$:
  - moves freely across sectors
  - is accumulated dynamically, but we treat it here as a parameter.
- Human Labor: initial allocation $\{\bar{h}_j\}$, where
  - $j \in [0,1] \cup r \cup e \cup \text{null}$.
  - Can move from j to j' by acquiring education $e_{jj'}$ at p_e per unit.
  - Cost to individual is $p_e e_{jj'}$.
  - Nests fully mobile labor $e_{jj'} = 0$ or fully specific labor $e_{jj'} = \infty$ for $j \neq j'$.

Prices and Unit Costs

- Prices:
  - Numeraire: return to non-robot physical capital;
  - $\mathbf{p} = (\{p_i\}$: the price system for final goods,
  - $(\mathbf{p}, p_r, p_e)$: the entire price system for final goods, robots and education;
  - $\mathbf{w} = (\{\omega_i\}, \omega_r, \omega_e)$: the wage system (labor could be partially or fully immobile).
- Output and input prices are connected by:

$$p_j = c_j(1, \nu_j),$$

  - where $\nu_j \equiv \min\{\omega_j, \lambda_j p_r\}$ is the effective price of the labor input, and
  - c_j is unit cost function: dual to the production function f_j.

The Markets for Humans, Robots and Physical Capital

- labor demand (human or non-human) must satisfy

$$v_j = p_j \frac{\partial f_j(k_j, \ell_j)}{\partial \ell_j} = \alpha_j(v_j) \frac{p_j y_j}{\ell_j},$$

where $\alpha_j(v_j)$ is output elasticity with respect to labor at labor price v_j.

- If $v_j = \omega_j < \lambda_j p_r$, then j is non-automated, and so:

$$h_j = \alpha_j(v_j) \frac{p_j y_j}{\omega_j} \text{ and } r_j = 0.$$

- If $v_j = \lambda_j p_r < \omega_j$, then j is fully automated, and so:

$$h_j = 0 \text{ and } r_j = \alpha_j(v_j) \frac{p_j y_j}{p_r}.$$

- Split in any way for "partially automated sectors" where $v_j = \lambda_j p_r = \omega_j$.

The Markets for Humans, Robots and Physical Capital, contd.

- Market clearing for humans:

  - Given **w** and initial and final human allocations $\{\bar{h}_j\}$ and $\{h_j\}$, no one wants to (or can) switch sectors.

- Market clearing for robots:

  - $y_r = \int_0^1 r_i di + r_r + r_e.$

- Market clearing for physical capital:

  - $\bar{K} = \int_0^1 [1 - \alpha_i(v_i)] p_i y_i di + [1 - \alpha_r(v_r)] p_r y_r + [1 - \alpha_e(v_e)] p_e y_e,$

  because the demand k_j equals $[1 - \alpha_j(v_j)] p_j y_j$ in every sector j.

Preferences, National Income, and Final Goods Markets

- $U(\mathbf{y})$: common utility function for all households.
  - Assume homothetic.
  - $\beta_i(\mathbf{p})$: expenditure share on good i, independent of total expenditure.
  - $\beta_i(\mathbf{p}) > 0$ for all continuous, bounded $\mathbf{p} \gg 0$ (and continuous in $\mathbf{p}$ in sup norm).
- National income given by

$$Y = \bar{K} + \int_0^1 \omega_i h_i + \omega_r h_r + \omega_e h_e$$

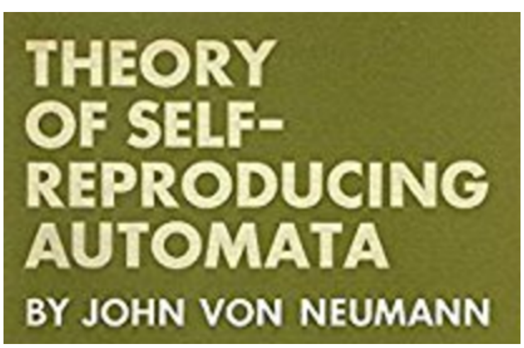
  - so each final goods market i clears when:

$$p_i y_i = \beta_i(\mathbf{p})[Y - p_e y_e].$$

  - obvious amendment when there is saving.

Von Neumann Singularity

- The case in which the robot sector is itself automated [$\lambda_r p_r < \omega_r$] is of special interest:



"The accelerating progress of technology, and changes in the mode of human life, give the appearance of approaching some essential singularity in the history of the race beyond which human affairs, as we know them, can not continue."

Von Neumann Singularity and Automation

- Proposition 1. Suppose that the following condition holds:

[S] $$\lambda_r \lim_{\rho \to 0} c_r(\rho, 1) < 1.$$

- Then there is $K^* > 0$ such that for all $\bar{K} \geq K^*$, the relative price of robots to capital is pinned at p_r^* *independent of* $\bar{K}$, and the robot sector becomes automated.

- Moreover, as $\bar{K} \to \infty$, every final goods sector becomes automated as well.

- If [S] fails, then the robot sector remains protected for all $\bar{K}$, and in general, so do some final goods sectors.

- For instance, if there is costless mobility of labor across sectors, then all goods j with $\lambda_j \geq \lambda_r$ are also protected irrespective of the value of $\bar{K}$.

Singularity

[S] $$\lambda_r \lim_{\rho \to 0} c_r(\rho, 1) < 1$$

- Pertains to just one sector, but implies automation in all sectors.

- Say $f_r(k, \ell) = \left[a_k k^{\frac{\sigma-1}{\sigma}} + a_\ell \ell^{\frac{\sigma-1}{\sigma}}\right]^{\sigma/(\sigma-1)}$ with $(a_k, a_\ell) \gg 0$ and $a_k + a_\ell = 1$.

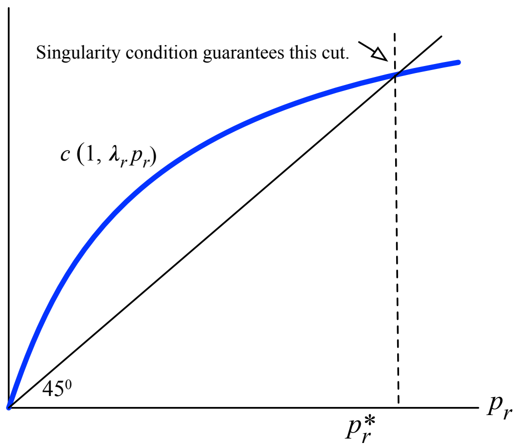
- Then $c_r(\rho, v) = \left[a_k^\sigma \rho^{1-\sigma} + a_\ell^\sigma v^{1-\sigma}\right]^{1/(1-\sigma)}$, so that [S] reduces to:

[S]$^{\text{CES}}$ $$\lambda_r \lim_{\rho \to 0} \left[a_k^\sigma \rho^{1-\sigma} + a_\ell^\sigma\right]^{1/(1-\sigma) < 1}.$$

- Automatically satisfied if $\sigma \geq 1$. Otherwise $\sigma \in (0, 1)$ and $\lambda_r < a_\ell^{\sigma/(\sigma-1)}$.

Automation Under [S]

- When [S] is satisfied, robot prices are bounded (relative to capital):
  - We always have $p_r \leq c_r(1, \lambda_r p_r)$, and in addition:



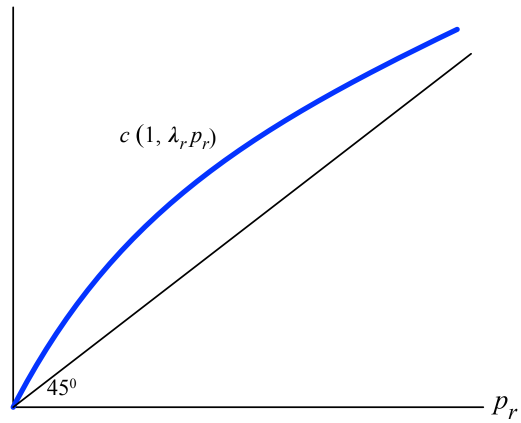
- In fact, when automated, $p_r = p_r^*$ independent of $\bar{K}$ (nonsubstitution theorem).

Automation Under [S], contd.

- Suppose that a positive measure of final sectors non-automated.
  - Then average labor supply to non-automated sectors is bounded.
- $p_j \leq c_j(1, \nu_j) \leq c_j(1, \lambda_j p_r) \leq c_j(1, \lambda_j p_r^*)$ [S] used here!, $\inf_{i,\bar{K}} p_j > 0$.
  - Therefore $\inf_j \beta_j(\mathbf{p}) > 0$ independent of j and $\bar{K}$.
- So as $\bar{K} \to \infty$, demand $\to \infty$ and $\omega_j \to \infty$ for each non-automated good;
  - But p_r is bounded, which contradicts the no-automation condition $\omega_j < \lambda_j p_r$.
- Now we can conclude that robot demand goes to infinity:
  - $\Rightarrow$ automation of robot sector by the same argument.
- Remark on automation in the education sector.

Converse: When [S] Fails

- If [S] fails, $c_r(1, \lambda_r p_r)$ never intersects p_r:



  - So $p_r < c_r(1, \lambda_r p_r)$ *always*: robot sector *never* automated.

- As $\bar{K}$ rises, human wages and robot prices rise in tandem.

  - So other final goods sectors will generally remain non-automated.

Singularity and Factor Shares

- All shares can be ultimately traced to capital and human labor

  - E.g., robot share paid out to capital, labor (and maybe robots).

  - With singularity, human labor also drops out:

- Proposition 2. Assume [S]. Then as $\bar{K} \to \infty$, the share of physical capital in national income converges to 1, and that of human labor converges to zero.

The Share of Capital Under [S]

- Recall market clearing for physical capital:
  - $\bar{K} = \int_0^1 [1-\alpha_i] p_i y_i di + [1-\alpha_r] p_r y_r + [1-\alpha_e] p_e y_e,$

  where we remember that all α's are endogenous.

The Share of Capital Under [S]

- Recall market clearing for physical capital:
  - $\frac{\bar{K}}{Y} = \frac{1}{Y} \int_0^1 [1-\alpha_i] p_i y_i di + [1-\alpha_r] \frac{p_r y_r}{Y} + [1-\alpha_e] \frac{p_e y_e}{Y},$

  where we remember that all α's are endogenous.

The Share of Capital Under [S]

- Recall market clearing for physical capital:
  - $$\frac{\bar{K}}{Y} = \frac{Y - p_e y_e}{Y} \int_0^1 [1-\alpha_i]\beta_i di + [1-\alpha_r]\frac{p_r y_r}{Y} + [1-\alpha_e]\frac{p_e y_e}{Y},$$

  where we remember that all α's and β's are endogenous.
- $$\lim_{\bar{K}\to\infty} \frac{p_r y_r}{Y} = \frac{Y - p_e y_e}{Y} \int_0^1 \alpha_i \beta_i di + \lim_{\bar{K}\to\infty} \alpha_e \frac{p_e y_e}{Y} + \lim_{\bar{K}\to\infty} \alpha_r \frac{p_r y_r}{Y}.$$

The Share of Capital Under [S]

- Recall market clearing for physical capital:
  - $$\frac{\bar{K}}{Y} = \frac{Y - p_e y_e}{Y} \int_0^1 [1-\alpha_i]\beta_i di + [1-\alpha_r]\frac{p_r y_r}{Y} + [1-\alpha_e]\frac{p_e y_e}{Y},$$

  where we remember that all α's and β's are endogenous.
- $$\lim_{\bar{K}\to\infty} [1-\alpha_r]\frac{p_r y_r}{Y} = \frac{Y - p_e y_e}{Y} \int_0^1 \alpha_i \beta_i di + \lim_{\bar{K}\to\infty} \alpha_e \frac{p_e y_e}{Y}.$$
  - Combine to get
  $$\lim_{\bar{K}\to\infty} \frac{\bar{K}}{Y} = 1.$$
  - (a.k.a. "reduction to dated quantities of capital")
- Remark: nothing to do with $r > g$.

Capital Accumulation and Wages

- The overall wage level must *rise* with the extent of automation:
  - It is precisely rising wages that contains the seeds of an ever-rising capital share.
- $\lambda^* p_r^* \leq$ highest limit wage $\leq \max\{\lambda^* p_r^*, \lambda_e p_r^*\}$
  - where $\lambda^* = \max\{\lambda_1, \lambda_r\}$.
- With free human mobility, the limit is precisely $\lambda^* p_r^*$.
  - This value could be high, or even unbounded.
  - That said, all bets are off if the automated cost of robot production drops below human subsistence.

Miscellaneous Remarks

- Is automation monotone in $\bar{K}$? Sometimes, but not always.
  - Assume free mobility with common wage ω
  - Think of λ_0 as small (or automation-prone) and λ_1 as very large (or protected).
  - Assume preferences and technology are both Cobb-Douglas.
- Proposition 3. Under these conditions:

(I) [S] is automatically satisfied, so universal automation occurs as $\bar{K} \to \infty$.

(II) Both automation and the wage rate are monotone increasing in capital.

(III) *At the same time*, capital accumulation monotonically increases the share of capital in national income.

Miscellaneous Remarks

- *Full* automation? Seriously?

We assumed that everything can be "ultimately contested" by robots. The accumulation of capital relative to humans does the rest. Hopefully (serious) poetry and some professional human interactions will never be automated, but don't bet on it.

- Why is capital accumulation not explicitly modeled?

It could be, with not much added. We need $\bar{K} \to \infty$ though (relative to humans).

- Which sectors get automated first?

With free mobility of humans, $\omega_j = \omega$ for all j, so automation varies inversely with λ_j (purely technological).

With education costs, unskilled sectors could be severely contested, so automation may take out the middle-skilled sectors, leaving highly skilled *and unskilled* sectors provisionally protected.

- What about directed technical progress? Will it come to the rescue of humans?

Don't think so. Not if labor is progressively getting more expensive relative to capital. However, there will be a demand for labor-productivity enhancements in currently protected sectors that could move things the other way, at least temporarily.

- Maybe new goods and tasks can only be performed by humans, at least to start with?

Again, don't bet on it. That could happen, and with some luck it could happen forever. But a social need does not translate into a market demand for such jobs/goods.

Closing Remarks: Capital's Share and Personal Income Distribution

- We described a theory of automation without technical progress:
  - Driven entirely by inevitable changes in the relative price of labor to capital.
- Condition [S] determines whether the price of robots will be more closely tied to capital, or to labor.
  - If it holds, automation is inevitable, and the functional share of capital goes to one . . .
  - . . . despite the fact that the wage rate must consistently rise over time (slow automation).

Implications for the Personal Distribution of Income

- Our analysis is silent on the personal distribution of income.
  - *That* will depend on how much individuals will save, and
  - In what *form* they will save.
- I'm pessimistic about the prospects of intelligent, informed savings in equity.
  - But this is the only way to rule capital as capital rules the earth.
- The alternative is to have the State do this for us.
  - Universal basic share [UBS], see Ray (2017) and Moene and Ray (2017).
  - Discussion: Why UBS is better for indexing, growth sharing, and whether it can be funded by a % acquisition of each new issue of corporate shares.