

# Lectures on Economic Inequality

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- [Inequality and Divergence I. Personal Inequalities](#), Slides 1 and 2
- Inequality and Divergence II. Functional Inequalities
- Inequality and Conflict I. Polarization and Fractionalization
- Inequality and Conflict II. Some Empirical Findings
- Inequality and Conflict III. Towards a Theory of Class Conflict

## Slides 2. Markets and Personal Inequality

- Two views on the evolution of inequality:
- **Equalization:** Inequality an ongoing battle between convergence and “luck.”
  - Solow 1956, Brock-Mirman 1972, Becker-Tomes 1979, 1986, Loury 1981...
- **Disequalization:** Markets intrinsically create and maintain inequality.
  - Ray 1990, 2006, Banerjee-Newman 1993, Galor-Zeira 1993, Ljungqvist 1993, Freeman 1996, Mookherjee-Ray 2000, 2010. . .

## Standard Accumulation Equations

Becker-Tomes 1979, Loury 1981

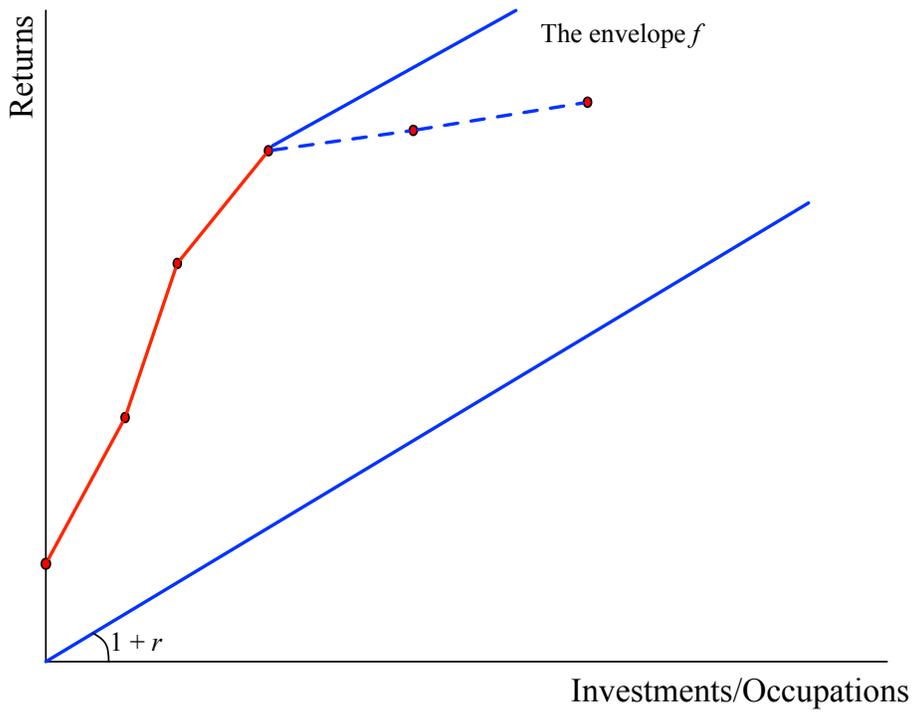
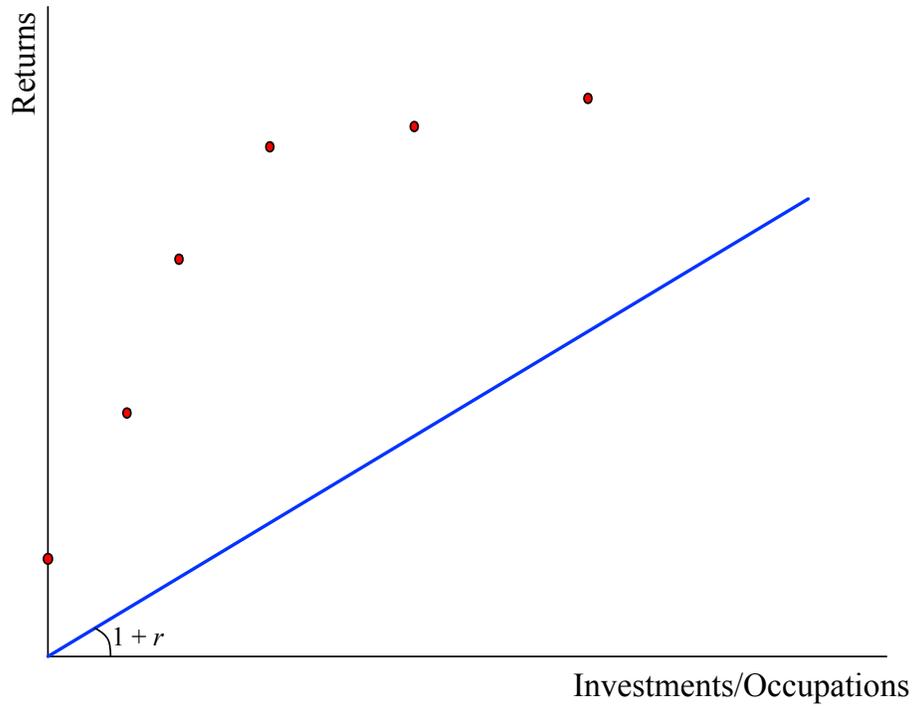
$$y_t = c_t + k_t,$$

income            consumption            investment/bequest

- **production function:**  $y_{t+1} = f(k_t)$  or  $f(k_t, \alpha_t)$ .
- Not surprising that this literature looks like growth theory.
  - Lots of “mini growth models”, one per household.
  - But it’s more than that, as we shall see.

## Interpreting $f$

- Standard production function as in growth theory
- Competitive economy:  $f(k) = w + (1 + r)k$ .
- Returns to skills or occupations: for example,
$$f(k) = \underline{w} \text{ for } k < \bar{x}$$
$$= \bar{w} \text{ for } k > \bar{x}.$$
  - May be exogenous to individual, but endogenous to the economy
  - So interpret  $f$  as envelope of intergenerational investments:
    - Financial bequests
    - Occupational choice



## Parental Preferences and Limited Mobility

- Parental utility  $U(c) + W(y')$ , where:
  - $U$  increasing and strictly concave, and  $W(y')$  increasing in progeny income  $y'$ .

$$\begin{array}{ccccc} W(y') & = & \delta[\theta V(y') & + & (1 - \theta)P(y')] \\ \text{Future utility} & & \text{Bellman value} & & \text{Exogenous value} \end{array}$$

- “Reduced-form” maximization problem:

$$\max U(c) + E_{\alpha} W(f(k, \alpha)).$$

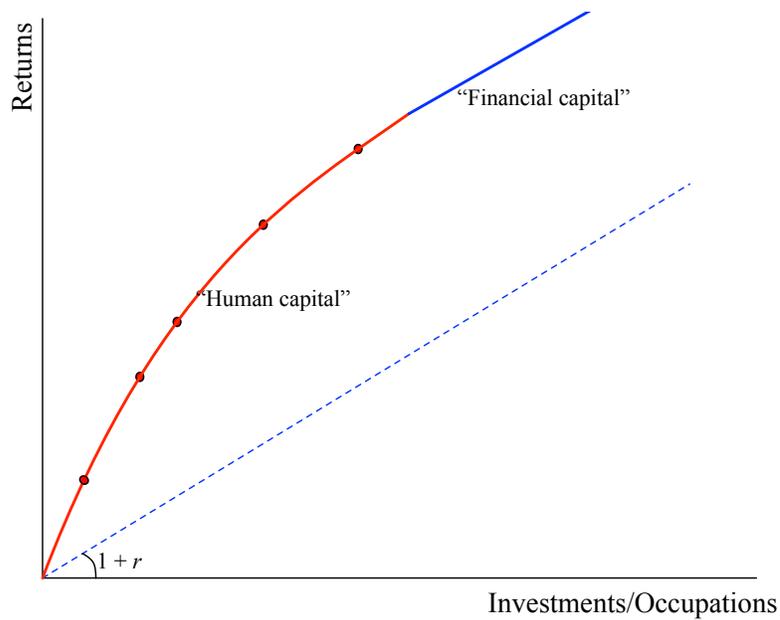
### ■ Theorem.

- Let  $h$  describe all optimal choices of  $k$  for each  $y$ .
- Then if  $y > y'$ ,  $k \in h(y)$ , and  $k' \in h(y')$ , it must be that  $k \geq k'$ .

### ■ Remarks:

- $h$  is “almost” a function.
- $h$  can only jump up, not down.
- Same assertion is not true of optimal  $c$ .
- Note how curvature of  $U$  is important, that of  $W$  is unimportant.
- Crucial for models in which  $f$  is endogenous with uncontrolled curvature.

**Standard Assumption:**  $f$  is *exogenous*, and typically *assumed* concave:

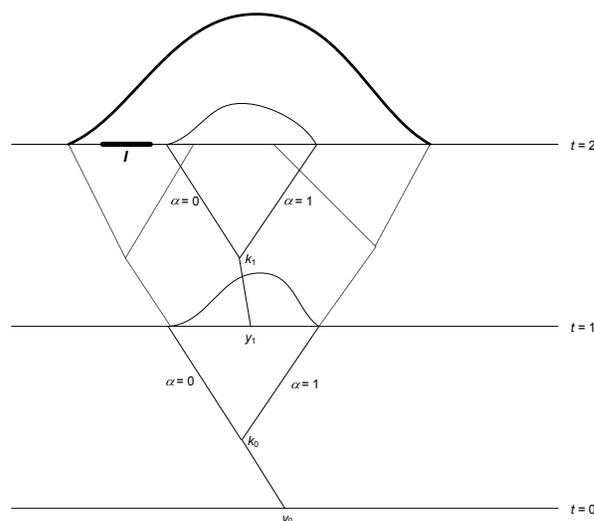


- Generates convergence to unique steady state in the absence of uncertainty.
- But won't be needed in the theorem I state next.

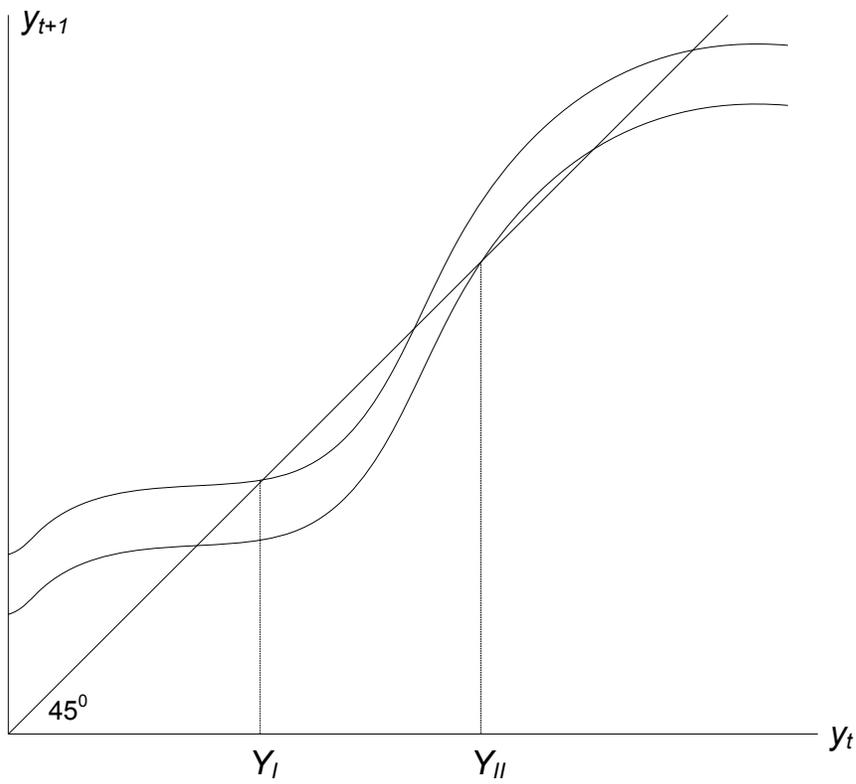
■ **Theorem** Brock-Mirman 1976, Becker-Tomes 1979, Loury 1981, but no concavity

■ Assume a *mixing condition*, such as  $f(0, 1) > 0$  (**poor genius**) and  $f(k, 0) < k$  for all  $k > 0$  (**rich fool**).

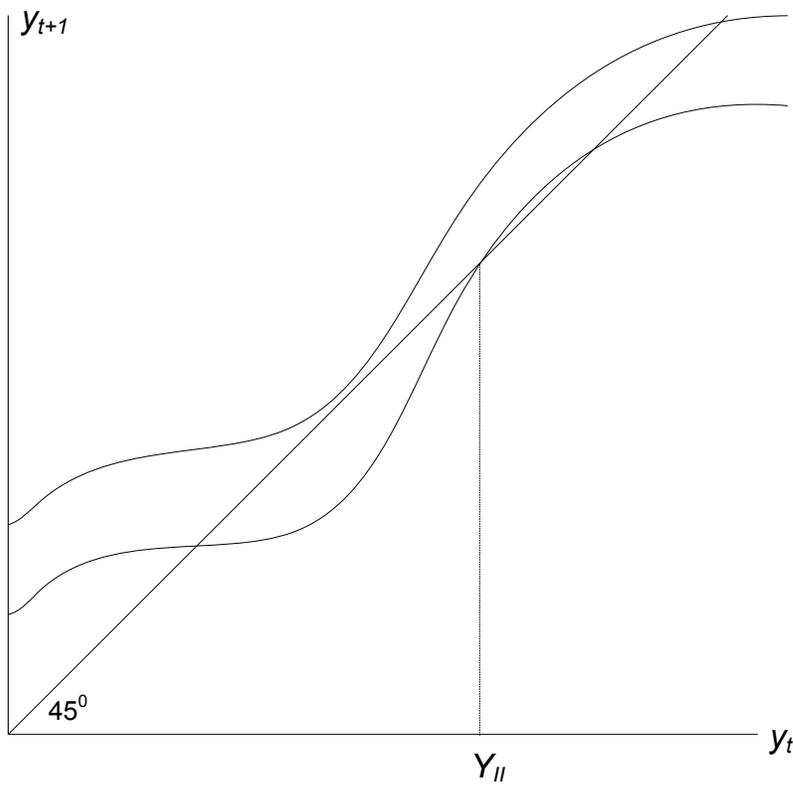
■ Then there exists a unique measure on incomes  $\mu^*$  such that  $\mu_t$  converges to  $\mu^*$  as  $t \rightarrow \infty$  from every  $\mu_0$ .



■ Core assumption: a “mixing zone”. In this case, it fails:



■ Here's a case where a mixing zone exists:



- Three major drawbacks of this model:
  - I. The reliance on stochastic shocks
    - Participation in national lottery  $\Rightarrow$  mixing.
    - Ergodicity could be a misleading concept.
    - Convexities and non-convexities all lumped together (*S-shaped example*).
  - II. No mixing condition  $\Rightarrow$  multiple steady states:
    - But must have *disjoint supports*, which is weird.
  - III. The *reliance on efficiency units*:
    - No way to endogenize the returns to different occupations.
    - Can't ask the question about how rates of return vary with wealth.

## Inequality and Markets

- Return to the interpretation of  $f$  as occupational choice.
  - Dropping efficiency units creates movements in relative prices:
    - $f$  isn't "just technology" anymore.
  - *An Extended Example* with just two occupations
    - Two occupations, skilled  $S$  and unskilled  $U$ . Training cost  $x$ .
    - Population allocation  $(\lambda, 1 - \lambda)$ .
    - Output:  $f(\lambda, 1 - \lambda)$
    - Skilled wage:  $w_s(\lambda) \equiv f_1(\lambda, 1 - \lambda)$
    - Unskilled wage:  $w_u(\lambda) \equiv f_2(\lambda, 1 - \lambda)$

## Households

- Continuum of households, each with one agent per generation.
- Starting wealth  $y$ ;  $y = c + k$ , where  $k \in \{0, x\}$ .
- Child wealth  $y' = w$ , where  $w = w_s$  or  $w_u$ .
- Parent maxes  $U(c) + \delta V(y')$  (Bellman equation)
- No debt!
- Child grows up; back to the same cycle.

## Equilibrium

- A sequence  $\{\lambda^t, w_s^t, w_u^t\}$  such that
- $w_s^t = w_s(\lambda^t)$  and  $w_u^t = w_u(\lambda^t)$  for every  $t$ .
- $\lambda^0$  given and the other  $\lambda^t$ s agree with utility maximization.
- **Steady State:** *stationary* equilibrium with positive output and wages:

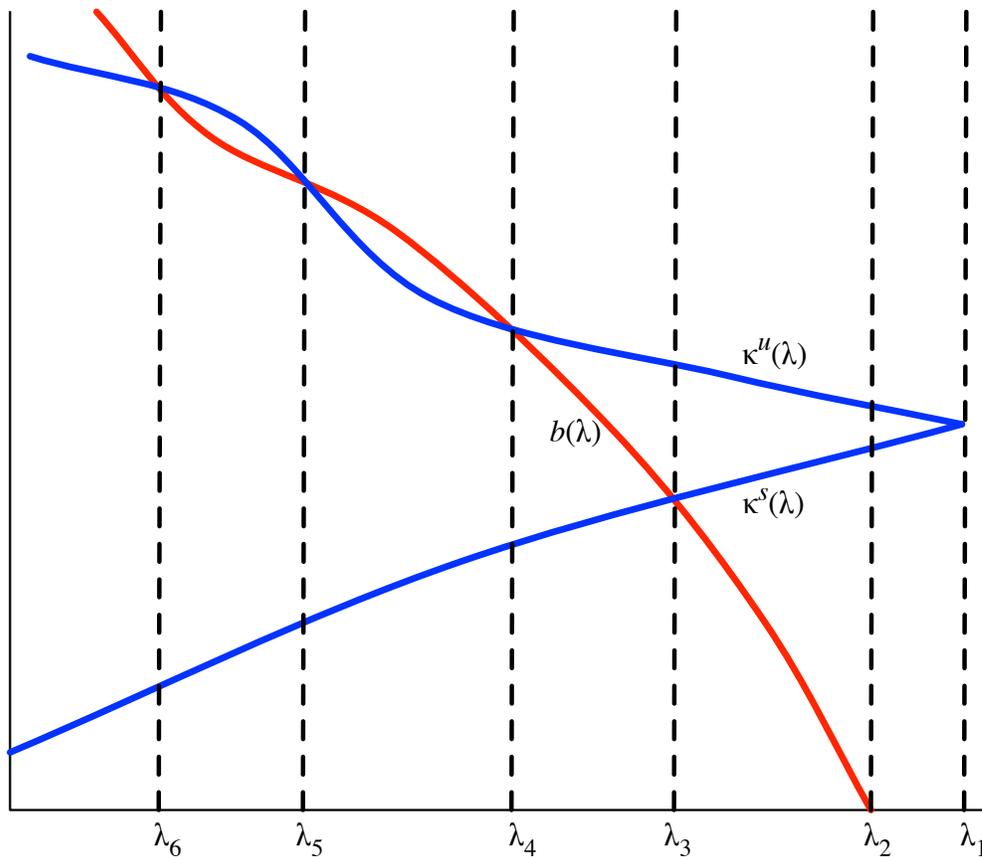
$$\kappa^u(\lambda) \geq b(\lambda) \geq \kappa^s(\lambda), \text{ where}$$

- $\kappa^j(\lambda) \equiv U(w_j(\lambda)) - U(w_j(\lambda) - X)$

(investment cost in utils)

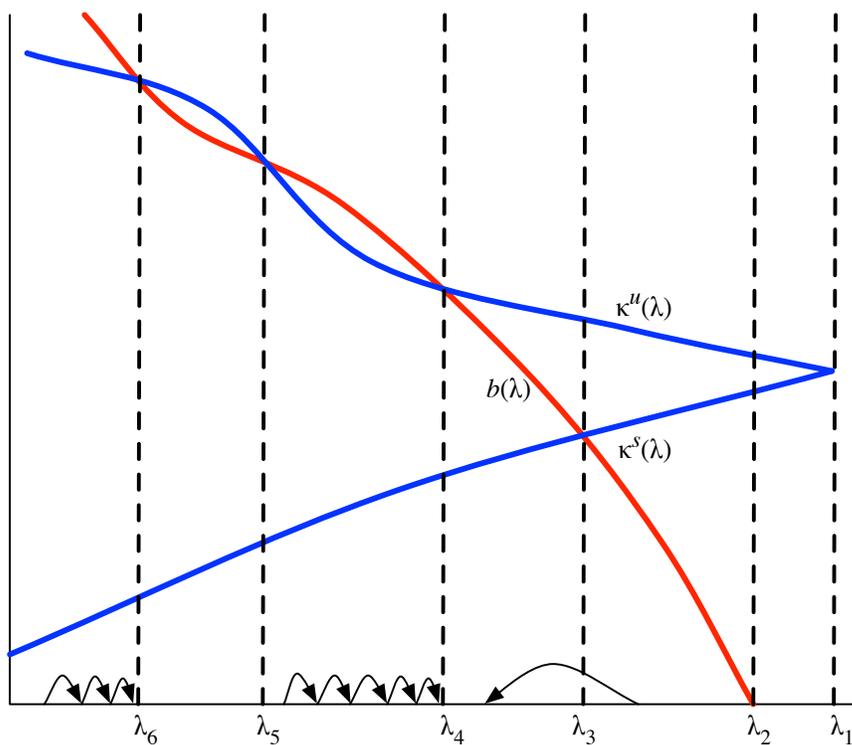
- $b(\lambda) \equiv V(w_s(\lambda)) - V(w_u(\lambda)) = \frac{1}{1-\delta} [u(w_s(\lambda) - X) - u(w_u(\lambda))]$

- (investment gain in utils)



- Two-occupation model useful for number of insights:
- **No convergence**; persistent inequality in **utilities**.
- Symmetry-breaking argument.
- **Multiple steady states** must exist.
- See diagram for multiple instances of  $\kappa^u(\lambda) \geq b(\lambda) \geq \kappa^s(\lambda)$ .
- Steady states with **less inequality** have **higher net output**.
- Net output maximization:  $\max_l F(l, 1 - l) - X$ . Say at  $l^*$ .
- So  $F_1(l^*, 1 - l^*) - F_2(l^*, 1 - l^*) = X$ .
- All steady states to left of this point: inequality  $\uparrow$ , output  $\downarrow$ .

- Can get an exact account of history-dependence (dynamics).



## Two Applications/Examples

### I. Industrialization with Fixed Costs

- Each person can set up factory at cost  $X$ .
- Gets access to production function  $g(L)$ , hire at wage  $w$ .
- Otherwise work as laborer.
- Multiple steady states in factory prevalence.

- To embed this story into two-occupation model:

- Define  $u =$  laborer,  $s =$  entrepreneur. Let

$$f(l, 1-l) \equiv lg\left(\frac{1-l}{l}\right).$$

- Then

$$w^u(l) = f_2(l, 1-l) = g'\left(\frac{1-l}{l}\right) = w,$$

- and

$$w^s(l) = f_1(l, 1-l) = g\left(\frac{1-l}{l}\right) - \frac{1-l}{l}g'\left(\frac{1-l}{l}\right) = \text{profits}.$$

## II. Conditionality in Educational Subsidies

- Recall that higher  $\lambda$  associated with higher net output.

- So there is a role for educational subsidies.

- Assume all subsidies funded by taxing  $w_s$  at rate  $\tau$ .

- **Unconditional subsidies:** give to unskilled parents.

$$T_t = \frac{\lambda_t \tau}{1 - \lambda_t} w_s(\lambda_t).$$

- Add this to the unskilled wage:  $w_u(\lambda_t) + T_t$ .

- **Conditional subsidies:** give to all parents conditional on educating children.

$$Z_t = \frac{\lambda_t \tau}{\lambda_{t+1}} w_s(\lambda_t).$$

## ■ Theorem.

- With unconditional subsidies, every left-edge steady state declines, lowering the proportion of skilled labor and increasing pre-tax inequality, which undoes some or all of the initial subsidy.
- With conditional subsidies, every left-edge steady state goes up, increasing the proportion of skilled labor. In steady state, no direct transfer occurs from skilled to unskilled, yet unskilled incomes go up and skilled incomes fall.
- Conditional subsidies therefore generate superior macroeconomic performance (per capita skill ratio, output and consumption) and welfare (Rawlsian or utilitarian).

## Other Applications

- Trade theory in which autarkic inequality determines comparative advantage.
- The necessity of country-level specialization when national infrastructure is goods-specific.
- Fertility patterns in models of occupational choice.

## A General Model with Financial Bequests and Occupational Choice

- Why study this?
  - Financial and human bequests
    - No need for persistent inequality in two-occupation model
  - Rich occupational structure
    - Now the “curvature” of occupational returns is fully endogenous.
  - New insights
    - The exact nature of history-dependence

- Production with capital and “occupations”.
  - Population distribution on occupations  $\lambda$  (endogenous).
  - Physical capital  $k$ .
  - Production function  $y = F(k, \lambda)$ , CRS and strictly quasiconcave.
- Training cost function  $\mathbf{x}$  on occupations:
  - incurred up front.
  - parents pay directly, or bequeath and then children pay.

## Prices

- Perfect competition.
- Return on capital fixed at rate  $r$  (international  $k$ -mobility).
- Returns to occupational choice: “wage” vector  $\mathbf{w} \equiv \{w(n)\}$ .
- $\mathbf{w}$  endogenous, together with  $r$  supports profit-maximization.

## Households

- Continuum of households, each with one agent per generation.
- Starting wealth  $y$ ;  $y = c + b + x(n)$ .
- Child wealth  $y' = (1 + r)b + \mathbf{w}_{t+1}(n)$ .
- Parent picks  $(b, n)$  to max utility.
- No debt!  $b \geq 0$ .
- Child grows up; back to the same cycle.

## Preferences and Equilibrium

- Preferences: mix of income-based and nonpaternalistic

$$U(c) + \delta[\theta V(y') + (1 - \theta)P(y')]$$

- Equilibrium:

- Wages  $\mathbf{w}_t$ , value functions  $V_t$ , and occupational distributions  $\lambda_t$  such that at every date  $t$ :
  - Each family  $i$  chooses  $\{n_t(i), b_t(i)\}$  optimally
  - Occupational choices  $\{n_t(i)\}$  aggregate to  $\lambda_t$ ;
  - Firms willingly demand  $\lambda_t$  at prices  $(\mathbf{w}_t, r)$ .
  - **Note:** physical capital willingly supplied to meet any demand.

## Steady State

- A **stationary equilibrium** with positive output and wages:
  - $\mathbf{w}_t = \mathbf{w} \gg 0$ , and
  - $(k_t, \lambda_t) = (k, \lambda)$  for all  $t$ , and  $F(k, \lambda) > 0$ .

## Divergence and History: Going Deeper

- Two notions of history-dependence.
- Individual (household destinies depend on past events)
- Economy-wide (multiple **distributions** of wealth)
- Former endemic in this model. Latter is what we are after.
- Literature usually studies a small number of occupations (two).
- Steady-state conditions written as inequalities
- Multiplicities are endemic (as we've seen).

## Rich Occupational Structure

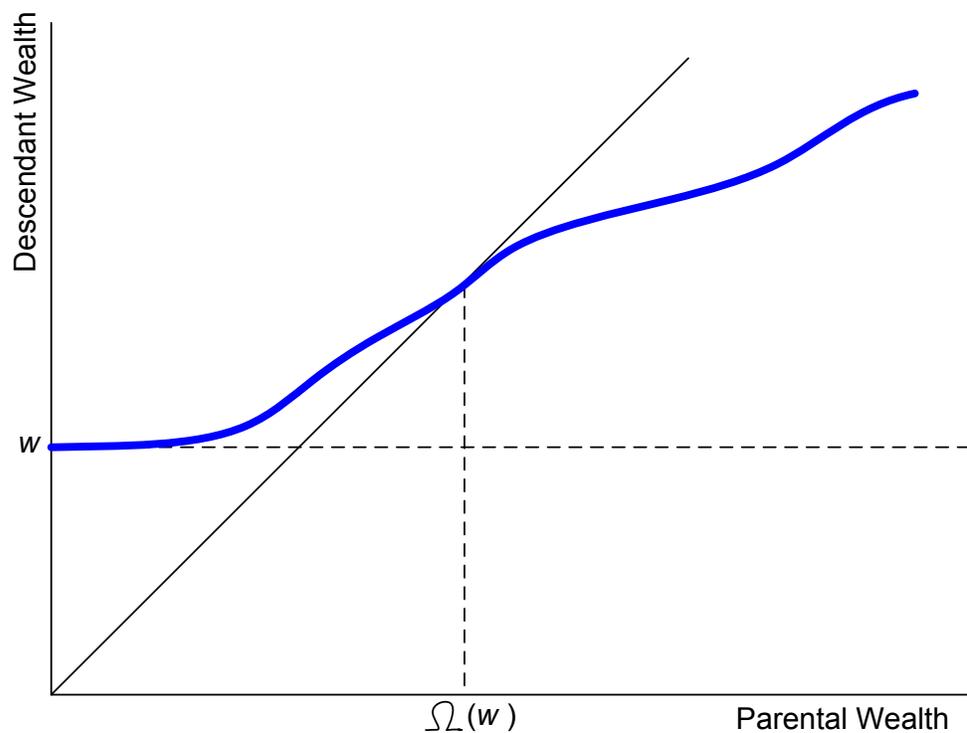
- Try the other extreme:
- The set of all training costs is a compact interval  $[0, X]$ .
- If  $\lambda$  is zero on any positive interval of training costs, then  $y = 0$ .
- Jointly the **richness** assumption [R].
- Want to investigate economy-wide history-dependence under this assumption.

## A Benchmark With No Occupational Choice

- Financial bequests (at rate  $r$ ) + just one occupation (wage  $w$ ).
- Parent with wealth  $y$  selects  $b \geq 0$  to

$$\max U(c) + \delta[\theta V(y') + (1 - \theta)P(y')].$$

- Child wealth  $y' \equiv w + (1 + r)b$ .
- Depends on  $(y, r, w)$ ; increasing in  $y$ .
- Limit wealth  $\Omega(w, r)$ : intersections with  $45^\circ$  line (or  $\infty$ ).
- [U]  $\Omega(\hat{w}, \hat{r})$  independent of initial conditions for all  $(\hat{w}, \hat{r})$ .
- [F]  $\Omega(\hat{w}, r) < \infty$  for all  $\hat{w}$ .

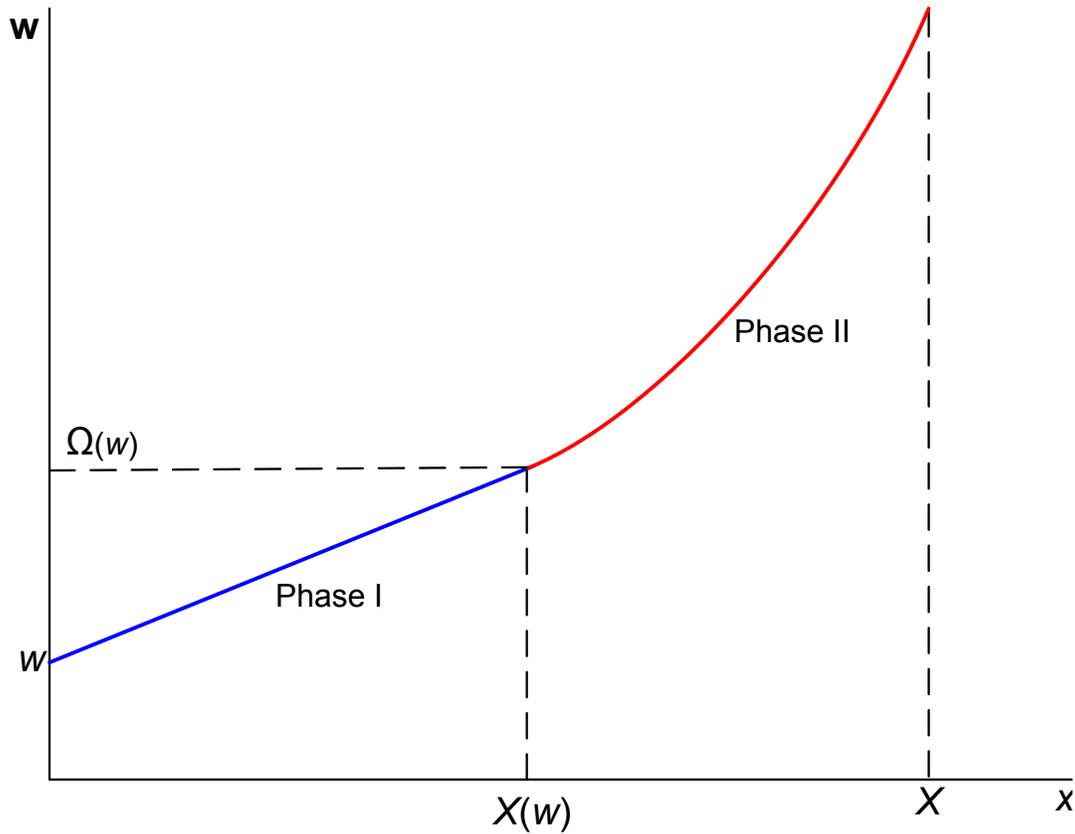


## Remarks on [U] and [F]

- Related to **limited persistence** (cf. Becker and Tomes).
- [U] requires some degree of paternalism in preferences:
  - Recall  $U(c) + \delta[\theta V(y') + (1 - \theta)P(y')]$
  - Need  $\theta < 1$ .
- Yet our results will generally extend to the dynastic case.

## Back to Occupational Choice

- **Theorem.** Assume [R], [U] and [F].
- Every steady state has wage function  $w$  continuous in  $x$ .
- $w$  is fully described by a **two-phase property**:



- In Phase I  $w$  is linear in  $x$ : there is  $w > 0$  such that

$$w(x) = w + (1 + r)x \text{ for all } x \leq \theta.$$

- All families in Phase I have the same **overall** wealth  $\Omega(w, r)$ .

- In Phase II,  $w$  follows the differential equation

$$w'(x) = \frac{U'(w(x) - x)}{\delta[\theta U'(w(x) - x) + (1 - \theta)P'(w(x))]}$$

- with endpoint to patch with I:  $w(x) = w + (1 + r)x$  at  $x = X(w)$ .

- Families located in Phase II will have different wealths.

$$w'(x) = \frac{U'(w(x) - x)}{\delta[\theta U'(w(x) - x) + (1 - \theta)P'(w(x))]}$$

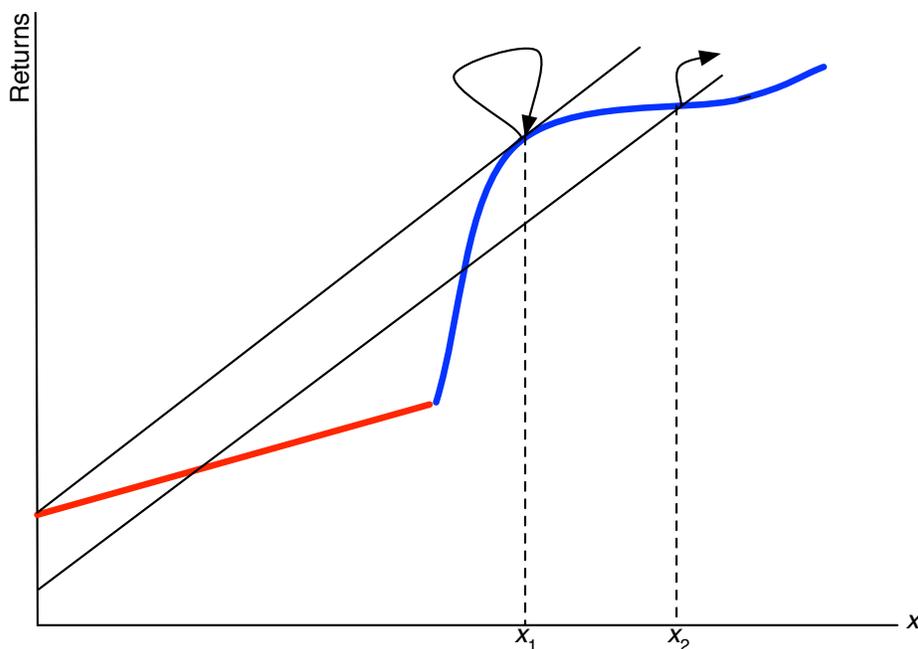
- Note that the shape of a steady state wage function
    - depends fundamentally on preferences
    - is independent of technology apart from baseline  $w$
- 

- Define the **average return** to occupational investment  $x$  by

$$\rho(x) \equiv \frac{w(x) - w}{x}.$$

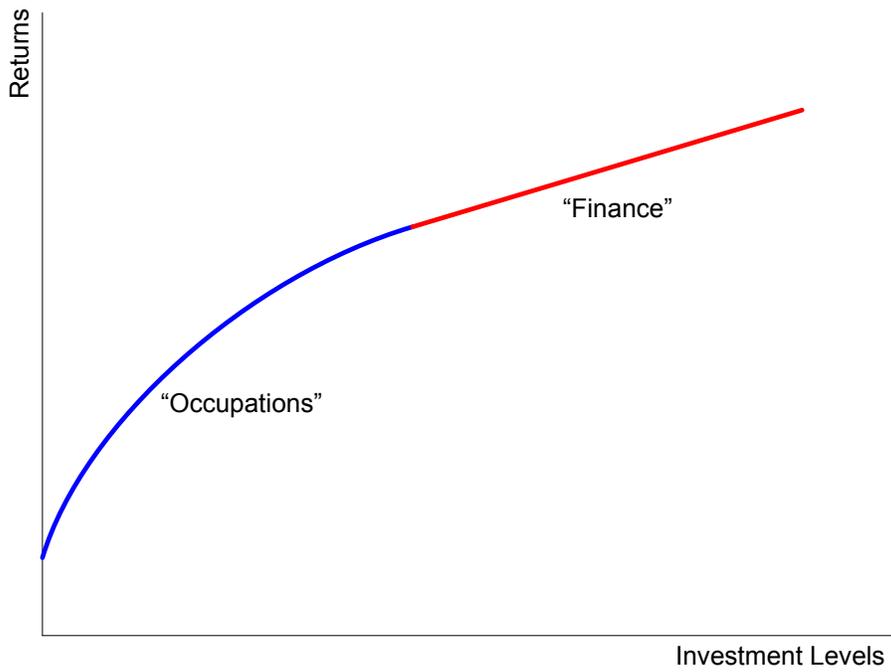
- **Theorem.** The average return to occupational investment is strictly increasing in  $x$  on  $[z, X]$ .

- **Proof.** Suppose not; then:

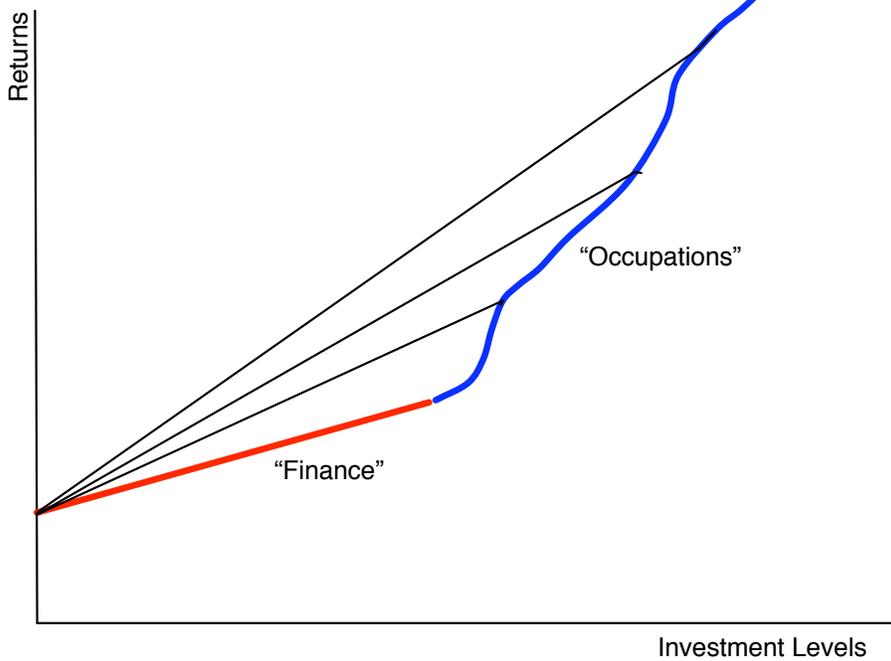


- Contradiction to unique limit wealth in the benchmark model.

- Theorem stands the usual literature on its head. Compare:



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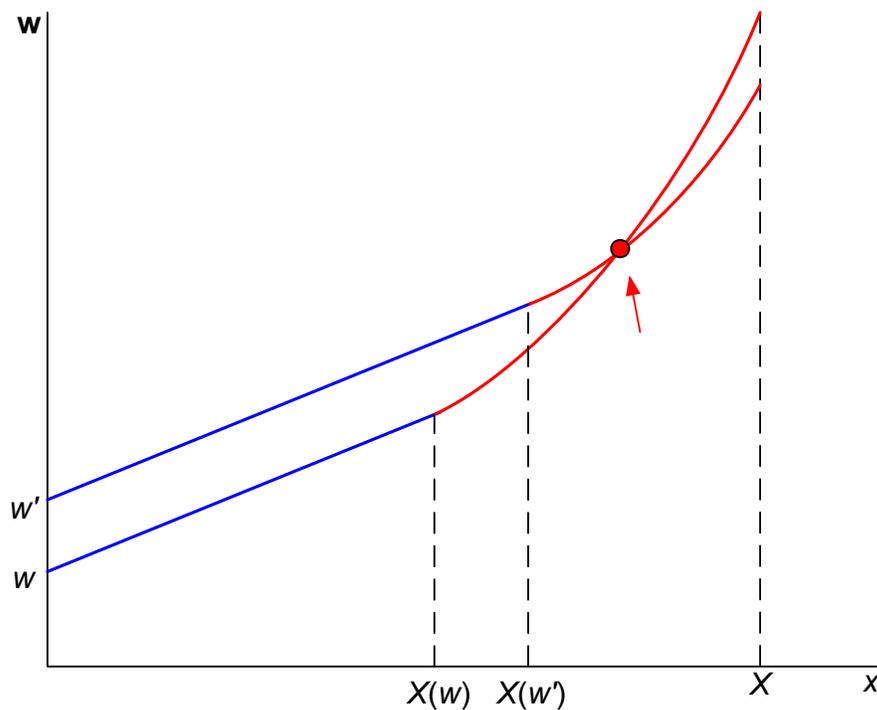


- Increasing occupational returns a (central) testable implication.

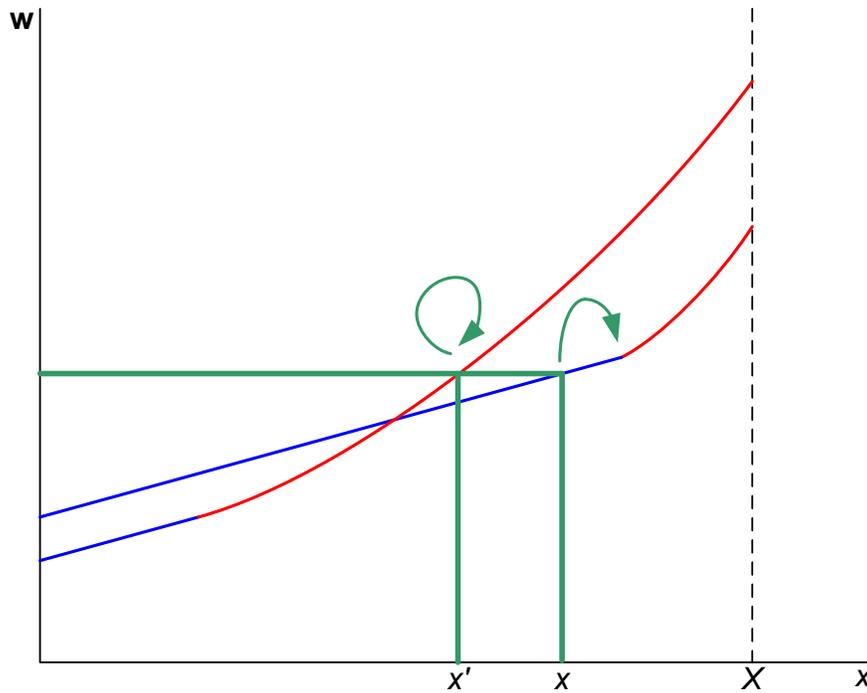
## Unique Steady State with Rich Occupational Structure

- Now a fundamental difference from two-occupation case:
- **Theorem.** Assume [R], [U] and [F]. Then there is at most one steady state.
- **Proof** rests on the fact that two members of the two-phase family cannot cross.
- See succeeding slides.
- Once that is settled, then only one intercept wage is possible that supports profit maximization with positive output.
- (For all wages must climb along with intercept wage.)

- No-crossing argument, part I
- Theory of differential equations won't allow this:



- No-crossing argument, part II
- Revealed preference argument rules this out:



### But What About Divergence?

- In Phase I, there is perfect equality of overall wealth.
- (All families in Phase I must have wealth equal to  $\Omega(w, r)$ .)
- Families at different occupations in Phase II **cannot** have the same wealth.
- Thus, “most” inequality comes from nonalienable capital.

“Labor income inequality is as important or more important than **all other income sources combined** in explaining total income inequality”.  
 [Fields (2004)]

- When is Phase II nonempty?
- When there is a large occupation span relative to bequest motive.

- Can examine this condition for different situations/applications.
- [Discounting](#).
- [Poverty](#), via TFP differences.
- [Growth](#) in TFP, lowers effective bequest motive
- [World return on capital](#).
- [Globalization](#): new occupations.

## [Divergence and History-Dependence](#)

- At the macro-level, history-dependence depends on occupational richness.
- A lot of history-dependence at the individual level.
- Individual dynasties have to occupy slots that are needed for aggregate production (or utility).
- Recall the world-economy interpretation, with individuals as countries.
- The distribution [as a whole](#) is pinned down, but not who occupies which slot.

## Luck versus Markets: Philosophy of Inequality

- **Equalization:** Inequality an ongoing battle between convergence and “luck”
- **Disequalization:** Markets intrinsically create and maintain inequality
- We’ve explored here the second approach, which:
  - (i) relies on symmetry-breaking to generate inequality in non-alienable activities.
  - (ii) is fundamentally interactive across agents (inequality is not the ergodic distribution of some isolated stochastic process).
  - (iii) generates new predictions for the curvature of the rate of return (and does not assume that curvature via efficiency units and an aggregate production function)
  - (iv) exhibits history-dependence at the level of individual dynasties, but less so at the macro level
- It remains to be seen if this is the right view of the world.