Lectures in Growth and Development

Ec428

(M. Ghatak, LSE, 2018)

Topic 2 : Growth Models

These notes are not guaranteed to be error free. If you spot one, please let me know.

Also material beginning with * means optional material.

Growth Models - Motivation

- Growth is essential for long-term improvements in standard of living

- For example, suppose we ask how much time it will take to double per capita income if an economy is growing at a rate g?

- Calculate

$$\frac{Y_t}{Y_0} = 2 = (1+g)^t \,.$$

- Or

$$\ln 2 = t \ln(1+g) \simeq tg$$

- As $\ln 2 \approx 0.7$, it will take 35 years if $g = 0.02$, 14 years if $g = 0.05$, and 7 years if $g = 0.1$

- The welfare implications are enormous!

- However, we should keep in mind that while growth is necessary for poverty alleviation or improvements in social indicators, it is not sufficient.

- For example, take the dream growth rate of 10%.

- It will take twenty-six years of sustained growth of 10% per year in incomes (no country in history has had a quarter century of sustained double digit growth!) to bring an Indian who is right on the poverty line up to merely the current level of per capita income, which is low by global standards to start with.

- Therefore, there is some argument in favour of redistributive policies to provide a minimum standard of living to the extreme poor

- Moreover, to take advantage of growth opportunities, the poor need access to human capital, the key inputs to which are education and health.

- Consider this fact: in India, the wage rate more than doubles if you move from low-skilled to medium-skilled jobs, or if you move from medium-skilled to high skilled jobs.

- If the child of an unskilled worker becomes highly skilled, then individual income will increase four-times within one generation.

- The key to sustained increases in standard of living is therefore to foster mobility through investments in human capital.

The Solow Model

- How does a person or economy grow richer?
  - You have some resources (skills, capital, land) which can be converted into output or income.
  - If you consume all your income in the current period, then clearly you cannot grow - at best you will be able to replicate what you did last period (i.e., provided the resources do not depreciate).
  - Savings, and investment are therefore key to growth.
  - Investment can be in physical and/or human capital (whether one's own or that of one's children)

- An institution-free world which is given by a single representative individual (Robinson Crusoe or a country's planner)

- Production function as a function of capital k_t (Fig. 1):

$$y_t = Ak_t^{\alpha}.$$

- Saves a constant fraction of his net investment so that capital next period is:

$$k_{t+1} = sy_t + (1 - \delta)k_t$$

or, $\triangle k_t = sy_t - \delta k_t$ where

$$\triangle k_t = k_{t+1} - k_t$$

and δ is the rate of depreciation.

- This defines a first-order non-linear difference equation in k (Fig. 2):

$$k_{t+1} = sAk_t^{\alpha} + (1 - \delta)k_t.$$

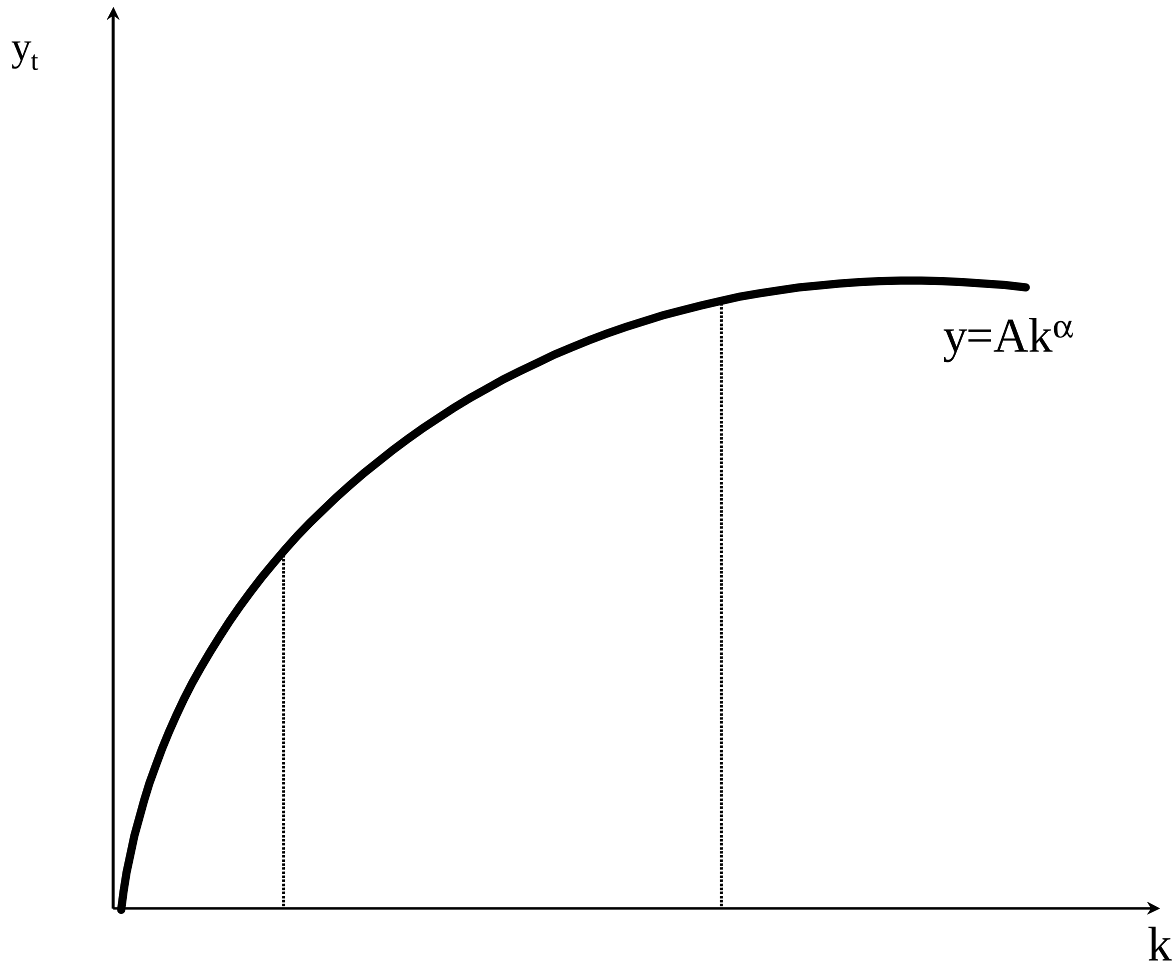

Figure 1: Production Function

Micro-foundations of Constant Saving Rate

- Preferences are homothetic and people save at a constant rate s, as in the Solow model.

- Alternatively, individuals live for one period, pass on a constant fraction s of their wealth as bequests to the next generation.

- Assume individuals have preferences over consumption and bequests given by:

$$U(c,b) = \log c + \beta \log b,\ \beta \geq 0.$$

- Maximize subject to $c + b \leq y$ and define $s \equiv \frac{\beta}{1+\beta}$

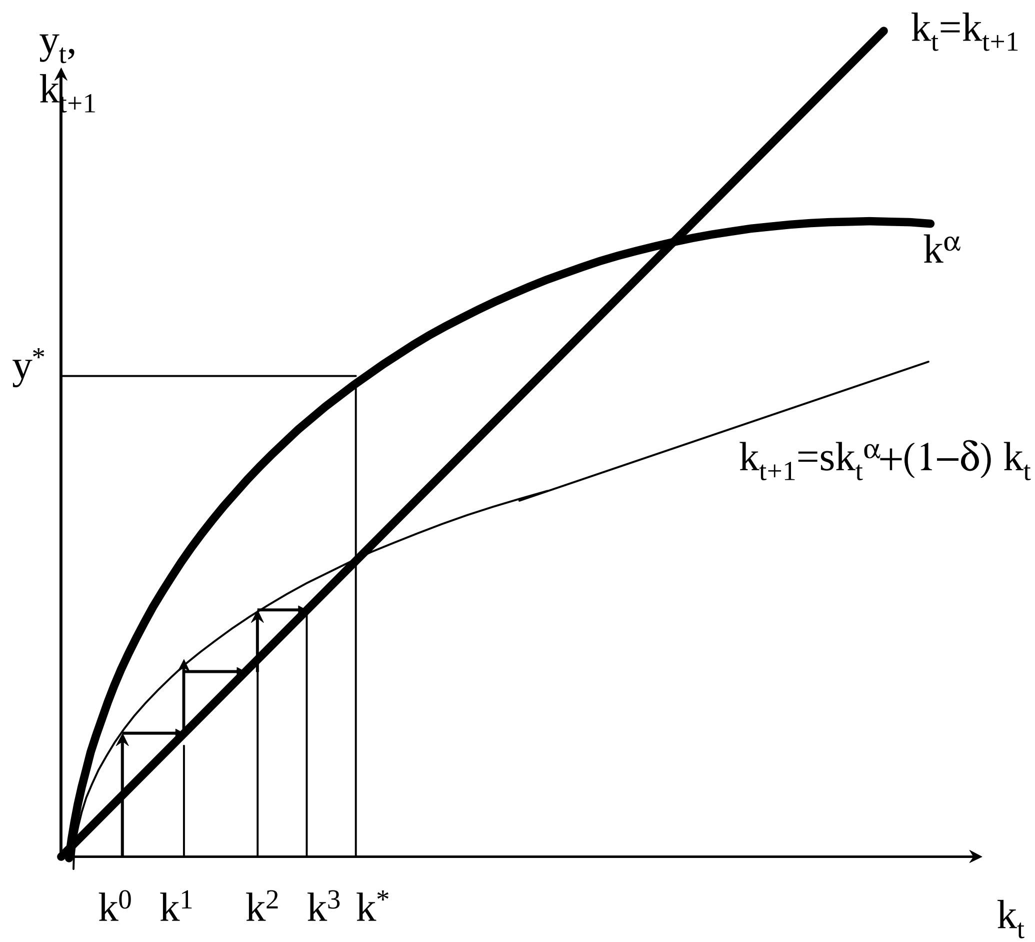

Figure 2: Convergence in Solow Model

- We will show later that we could alternatively derive it from the behaviour of forward-looking infinitely-lived decision maker under some conditions

The Steady State

- The formula for the steady state capital stock is :

$$k_{t+1} = k_t$$

or

$$\triangle k_t = sy_t - \delta k_t = 0$$

or

$$sy^* = \delta k^*$$

or

$$sA(k^*)^\alpha = \delta k^*.$$

- Solve for steady state level of capital stock and output

$$\begin{aligned} k^* &= \left(\frac{sA}{\delta}\right)^{\frac{1}{1-\alpha}} \\ y^* &= A^{\frac{1}{1-\alpha}} \left(\frac{s}{\delta}\right)^{\frac{\alpha}{1-\alpha}}. \end{aligned}$$

- Who will be richer in steady state? The model gives a simple answer : whoever has a higher value of s and A or a lower value of δ.

- The growth rate of the capital stock behaves in the following way:

$$\frac{\triangle k}{k} = sAk^{\alpha-1} - \delta$$

- Since $\alpha < 1$ the growth rate is declining in the level of the capital stock in the transition phase.

- Since

$$\log y = \alpha \log k + \log A$$

or,

$$\frac{\triangle y}{y} = \alpha \frac{\triangle k}{k}.$$

the same is true of per capita income. The poor grow faster.

The Neo-classical growth Model

- In the Solow model, the saving rate is exogenously given

- You save so that you can increase consumption in the future

- This choice is explicitly modeled in the neo-classical growth framework:

$$\max_{\{c_t,k_t\}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to $c_t + k_{t+1} \leq f(k_t) + (1-\delta)\, k_t$

- β is the discount factor, assumed to lie in $(0, 1)$

- k_0 is given

- The intertemporal budget constraint says total output can be either be consumed or saved as next period's capital stock

- Optimality condition known as the Euler equation:

$$u'(c_{t+1}) = \beta u'(c_t) \left\{ f'(k_{t+1}) + (1 - \delta) \right\}$$

- Nice interpretation: if you reduce consumption at the end of today by ε then you lose $u'(c_t)$ but this increases k_{t+1}, which generates an extra output of amount $f'(k_{t+1}) + (1 - \delta)$ at the end of next period, which you can consume and increase your utility by $u'(c_{t+1})$

- You weight this gain in tomorrow's consumption by β

- The Euler equation gives a second-order difference equation

- There exists a steady-state k^* and under standard assumptions, there is convergence starting with a given k_0

- Along the optimal path these must be equal

- In steady state $k_{t+1} = k_t = k_{t+2}$

- Therefore, the first-order condition gives us

$$1 = \beta \left\{ f'(k) + (1 - \delta) \right\}.$$

- We can solve k^* out explicitly if we are given $f(.)$ (e.g., k^α)

- Very similar to steady condition is Solow with s being an increasing function of β (more patient people have a higher saving rate).

- The Euler equation is

$$\frac{u'(c_t)}{u'(c_{t+1})} = \beta \left\{ f'(k_{t+1}) + (1 - \delta) \right\}$$

- As k_{t+1} rises along the transition path, $\frac{c_{t+1}}{c_t}$ rises

- For example, if $u(c_t) = c_t^{\gamma}$ then

$$\left(\frac{c_{t+1}}{c_t} \right)^{1-\gamma} = \beta \left\{ f'(k_{t+1}) + (1 - \delta) \right\}$$

- Defines a second-order difference equation in k

$$\left(\frac{f(k_{t+1}) + (1 - \delta)\, k_{t+1} - k_{t+2}}{f(k_t) + (1 - \delta)\, k_t - k_{t+1}} \right)^{1-\gamma} = \beta f'(k_{t+1})$$

- It converges given the assumption of diminishing returns

- In steady state (with $k_t = k^*$ for all t) $\frac{c_{t+1}}{c_t} = \beta f'(k^*) = 1$

- From the budget constraint

$$c_t + k_{t+1} = f(k_t) + (1 - \delta) k_t$$

- Therefore, in steady state we can solve for the level of consumption as well:

$$c^* = f(k^*) - \delta k^*.$$

Special Case that Yields a Constant Saving Rate

- Take the special case of $u(c_t) = \ln c_t$, $f(k_t) = k_t^\alpha$ and $\delta = 1$

- $\delta = 1$ refers to full depreciation - all capital is working capital

- The Euler equation is then:

$$\frac{c_{t+1}}{c_t} = \alpha\beta k_{t+1}^{\alpha-1}$$

- From the budget constraint $c_t + k_{t+1} = k_t^\alpha$

- Substituting we get

$$\frac{k_{t+1}^{\alpha} - k_{t+2}}{k_t^{\alpha} - k_{t+1}} = \frac{\alpha\beta}{k_{t+1}^{1-\alpha}}$$

- Simplifying, this yields

$$k_{t+1} = \frac{\alpha\beta}{1+\alpha\beta} k_t^{\alpha} + \frac{1}{1+\alpha\beta} \frac{k_{t+2}}{k_{t+1}^{\alpha}} k_{t+1}$$

- We claim that the saving rate is constant, namely

$$k_{t+1} = s k_t^{\alpha}$$

- Then substituting $k_{t+2} = sk_{t+1}^{\alpha}$ we get

$$k_{t+1} = \frac{\alpha\beta}{1+\alpha\beta} k_t^{\alpha} + \frac{1}{1+\alpha\beta} sk_{t+1}$$

- If we solve for s we get $s = \alpha\beta < 1$

Is Long Run Growth Possible?

- Take $u(c_t) = \ln c_t$, $f(k_t) = k_t^{\alpha} + \lambda k_t$ and $\delta = 1$ where $\lambda > 0$

- The Euler equation is then:

$$\frac{c_{t+1}}{c_t} = \beta \left(\alpha k_{t+1}^{\alpha-1} + \lambda \right)$$

- As $k_{t+1} \to \infty$, $\alpha k_{t+1}^{\alpha-1} \to 0$ but the marginal product of capital approaches λ

- If $\beta\lambda > 1$ then

$$\lim_{t \to \infty} \frac{c_{t+1}}{c_t} = \beta\lambda$$

- The condition for growth in the long-run is $\beta f'(k_{t+1}) > 1$ or, $f'(k_{t+1}) > \frac{1}{\beta}$

The Main Lessons

Lesson 1: Convergence. Being poor is no handicap in the long run. History does not matter.

Lesson 2: No long-run growth without constant returns or technological progress.

- The limits to growth comes from diminishing returns

- Reflects some fixed factor which slows down the growth rate (e.g. land, natural resources)

- People and economies reach their "steady states" where there is no growth barring shocks to technology or preferences.

- “Convergence” implication of growth models - applies both in time series and cross section.

- You grow faster when you are smaller but as you approach steady state, the growth rate slows down.

- If country A has more capital than country B, then it will grow slower. As the poor grow faster, they "catch up".

- Any persistent differences across countries must be pinned down differences in innate abilities of the people, its natural resources, attitudes regarding thrift, enterprise.

- One must have permanent policy measures in place (e.g., tax incentives to encourage savings) to do anything about it.

Relating the Solow Model to the "Growth Facts" of Topic 1

- Output is given by

$$Y = K^{\alpha} (AL)^{1-\alpha}.$$

- Let output per worker and capital per worker be denoted by:

$$y = \frac{Y}{L} \text{ and } k = \frac{K}{L}$$

- Define $\tilde{L} = AL$ as "efficiency units of labor".

- Let $L_t = L_0(1+n)^t$ where n is the exogenously given growth rate of population

- This yields

$$\frac{L_{t+1}}{L_t} = \frac{L_0(1+n)^{t+1}}{L_0(1+n)^t} = 1+n$$

- Let $A_t = A_0(1+g)^t$ which is labour-augmenting technological change

- This yields

$$\frac{A_{t+1}}{A_t} = \frac{A_0(1+g)^{t+1}}{A_0(1+g)^t} = 1+g$$

- Let δ be the rate of depreciation

- We have

$$\begin{aligned} Y &= K^{\alpha} AL AL^{-\alpha} \\ \frac{Y}{AL} &= \left(\frac{K}{AL}\right)^{\alpha} \end{aligned}$$

- Normalizing output and capital by the size of the effective labour force (AL) and denoting these by $\tilde{y} \equiv \frac{Y}{AL}$ and $\tilde{k} \equiv \frac{K}{AL}$ we can write the production funciton as

$$\tilde{y} = \left(\tilde{k}\right)^{\alpha}$$

- The capital accumulation equation is

$$
\begin{aligned}
K_{t+1} &= sY_t + (1-\delta) K_t \\
\frac{K_{t+1}}{A_{t+1}L_{t+1}} &= \left\{ s\frac{Y_t}{A_tL_t} + (1-\delta)\frac{K_t}{A_tL_t} \right\} \frac{A_tL_t}{A_{t+1}L_{t+1}} \\
\tilde{k}_{t+1} &= \left\{ s\tilde{y}_t + (1-\delta)\tilde{k}_t \right\} \frac{1}{(1+g)(1+n)} \\
&\approx \frac{s\tilde{y}_t + (1-\delta)\tilde{k}_t}{1+n+g}
\end{aligned}
$$

- Therefore,

$$
\triangle \tilde{k}_t = \tilde{k}_{t+1} - \tilde{k}_t = \frac{s\tilde{k}_t^{\alpha} - (n+\delta+g)\tilde{k}_t.}{1+n+g}
$$

- Steady state:

$$
\begin{aligned}
s\left(\tilde{k}^*\right)^{\alpha} &= (n+\delta+g)\,\tilde{k}^* \\
\tilde{k}^* &= \left(\frac{s}{n+\delta+g}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}
$$

- Steaty state per capita income in efficiency units

$$
\tilde{y}^* = \left(\tilde{k}^*\right)^{\alpha} = \left(\frac{s}{n+\delta+g}\right)^{\frac{\alpha}{1-\alpha}}.
$$

- We know why it is decreasing in δ. Why is it decreasing in n and g?

- The higher is n, there are more workers every period and so K has to go up to keep K/L steady and because of diminishing returns, this means the steady state level of K/L has to be lower

- The higher is g, there are more effective workers every period due to technological progress, and so K has to rise and by the same logic as in the previous bullet point, the steady state level of $\tilde{k}^*$ is lower

- In steady state

$$
\begin{aligned}
\tilde{y}_{t+1} &= \tilde{y}_t \\
\frac{Y_{t+1}}{A_{t+1}L_{t+1}} &= \frac{Y_t}{A_tL_t} \\
\frac{y_{t+1}}{y_t} &= \frac{A_{t+1}}{A_t} = 1+g
\end{aligned}
$$

- By the same logic

$$\frac{k_{t+1}}{k_t} = \frac{A_{t+1}}{A_t} = 1+g$$

- Therefore, both GDP per capita grows and capital per worker grows at the rate of technological progress in steady state (recall figures 1 and 2 from Jones, 2016).

- Now the capital output ratio is given by

$$
\begin{aligned}
\frac{K_{t+1}}{Y_{t+1}} &= \frac{k_{t+1}}{y_{t+1}} \\
&= \frac{k_t\,(1+g)}{y_t\,(1+g)} \\
&= \frac{k_t}{y_t} \\
&= \frac{K_t}{Y_t}.
\end{aligned}
$$

- The rate of growth of GDP per capita is constant - we showed that GDP per capita grows at the rate of technological progress and since we assume that technological progress takes place at a constant rate, then the rate of growth of GDP per capita is constant too.

- The ratio of the total capital stock to GDP is constant also follows from above

- Output is

$$Y = K^{\alpha} (AL)^{1-\alpha}$$

- Therefore, w and r are given by

$$\begin{aligned} w &= (1-\alpha) K^{\alpha} A^{1-\alpha} L^{-\alpha} \\ r &= \alpha K^{\alpha-1} A^{1-\alpha} L^{1-\alpha} \end{aligned}$$

- The share of labor income in GDP is given by

$$\frac{wL}{Y} = \frac{(1-\alpha)K^{\alpha}A^{1-\alpha}L^{-\alpha}L}{K^{\alpha}(AL)^{1-\alpha}} = (1-\alpha)$$

- This is constant, and so is the share of capital as they add up to 1

- This holds in steady state or along the transition path

- However, if we took a general CRS production function, this would still hold in steady state

- The reason is

$$
\begin{aligned}
\frac{wL}{Y} &= \frac{\frac{\partial F(K,AL)}{\partial L}L}{F(K,AL)} \\
&= \frac{AF_L(K,AL)L}{F(K,AL)} \\
&= \frac{AF_L(\frac{K}{AL},1)L}{F(\frac{K}{AL},1)AL} \\
&= \frac{F_L(\frac{K}{AL},1)}{F(\frac{K}{AL},1)}
\end{aligned}
$$

- This is indeed a constant in steady state

- The step that $F_L(K, AL) = F_L(\frac{K}{AL}, 1)$ follows from the CRS assumption $F(\lambda K, \lambda AL) = \lambda F(K, AL)$ and taking the derivative with respect to L on both sides, getting $F_L(\lambda K, \lambda AL)\lambda A = \lambda A F_L(K, AL)$ for all $\lambda > 0$

- The average rate of return on capital is constant follows from this model

- The net interest rate is

$$\begin{aligned} r &= F_K(K, AL) - \delta \\ &= F_K\left(\frac{K}{AL}, 1\right) - \delta \end{aligned}$$

- As $\frac{K}{AL}$ is a constant, so is r

- Therefore, the Solow model does well in terms of consistency with the stylized facts that Jones (2016) refers to, several of which go back to Kaldor (1957).

Some Simple Calibration Exercises

- Suppose now we take the Solow model seriously, and try to get a quantitative sense of how much parametric variation will generate large output differences

- Let us take two countries indexed by 1 and 2

- Then we have

$$\frac{y_1^*}{y_2^*} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{1-\alpha}} \left(\frac{s_1}{s_2}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\delta_2}{\delta_1}\right)^{\frac{\alpha}{1-\alpha}}.$$

- The effect of varying savings rates will only depend on α

- In a Cobb-Douglass world of perfect competition, α is the share of capital in national income, and Lucas (1990) estimates this at $\frac{1}{3}$

- Therefore $\frac{\alpha}{1-\alpha} = \frac{1}{2}$ and so doubling the savings rate (a big increase) will increase per capita income by $2^{\frac{1}{2}} \simeq 1.41$ which means around a 40% increase (not much)

- For differences in productivity (A) the difference is more amplified

- Doubling A will increase per capita income by $2^{\frac{3}{2}} \simeq 2.82$ which means close to a 200% increase, which is closer to what we see.

- Now consider comparing interest rates across countries.

- We assume a Cobb-Douglas production function $y = Ak^{\alpha}$ and so

$$r = \alpha A k^{\alpha - 1}$$

- If we compare two countries, 1 and 2 we have:

$$\begin{aligned}
\frac{r_1}{r_2} &= \frac{\alpha A k_1^{\alpha-1}}{\alpha A k_2^{\alpha-1}} \\
&= \left(\frac{k_1}{k_2}\right)^{\alpha-1}
\end{aligned}$$

- From the production function, $k = \left(\frac{y}{A}\right)^{\frac{1}{\alpha}}$ and so

$$\frac{r_1}{r_2} = \left(\frac{y_1}{y_2}\right)^{\frac{\alpha-1}{\alpha}}$$

- With the Cobb-Douglas production function, α corresponds to the capital-income share of GDP which is 0.35 or so for the US

- If we compare Mexico (1) and the US (2), the former's GDP per capita is approximately 0.3 times that of the latter and so

$$\frac{r_1}{r_2} = (0.3)^{\frac{0.35-1}{0.35}} \approx 9.35$$

- The actual gap between the interest rates is much smaller - it is 7.75% for Mexico and 2.25% for the US

- Also, the incentives for investors from rich countries to invest in poor countries would be huge (and that of investing in their own country very little).

Basic Growth Accounting Exercise

- Take a production function where Y is output, K is capital, and L is labour

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

- Taking logarithms

$$\ln(Y_t) = \ln(A_t) + \alpha \ln(K_t) + (1-\alpha)\ln(L_t)$$

- Let $g_X = \frac{1}{X_t}\frac{dX_t}{dt}$

- Differentiating with respect to t we get the following growth decomposition expression:

$$g_Y = g_A + \alpha g_K + (1 - \alpha)\, g_L.$$

- From Easterly and Levine (2001) we have the following summary table:

TABLE 1. Selected Growth Accounting Results for Individual Countries (percent)

Economy	Share of capital in national output	GDP growth	Share contributed by Capital	Labor	TFP
OECD 1947–73					
France	.40	5.40	41	4	55
Germany	.39	6.61	41	3	56
Italy	.39	5.30	34	2	64
Japan	.39	9.50	35	23	42
United Kingdom	.38	3.70	47	1	52
United States	.40	4.00	43	24	33
OECD 1960–90					
France	.42	3.50	58	1	41
Germany	.40	3.20	59	–8	49
Italy	.38	4.10	49	3	48
Japan	.42	6.81	57	14	29
United Kingdom	.39	2.49	52	–4	52
United States	.41	3.10	45	42	13
Latin America 1940–80					
Argentina	.54	3.60	43	26	31
Brazil	.45	6.40	51	20	29
Chile	.52	3.80	34	26	40
Mexico	.69	6.30	40	23	37
Venezuela	.55	5.20	57	34	9
East Asia 1966–90					
Hong Kong, China	.37	7.30	42	28	30
Singapore	.53	8.50	73	32	–5
Korea, Rep. of	.32	10.32	46	42	12
Taiwan, China	0.29	9.10	40	40	20

Source: For OECD, Christenson, Cummings, and Jorgenson (1980) and Dougherty (1991); for Latin America, Elias (1990); for East Asia, Young (1995).

- How to interpret the negative numbers?
  - Those factors expanded faster than growth of output and so were not effective (think of capital accumulation in a centrally planned economy or population growth in a labour surplus economy)

- The role of TFP would be higher if we looked at output per worker instead

Allowing for Human Capital

- Based on Jones (2016)

- Take a production function where Y is output, K is capital, and H is labour augmented by human capital

$$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

- Dividing by Y_t^{α}

$$Y_t^{1-\alpha} = A_t \left(\frac{K_t}{Y_t}\right)^{\alpha} H_t^{1-\alpha}$$

- Or,

$$Y_t = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} H_t$$

- Dividing both sides by L_t

$$\frac{Y_t}{L_t} = (A_t)^{\frac{1}{1-\alpha}} \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{L_t}.$$

- Taking logarithms

$$\ln\left(\frac{Y_t}{L_t}\right) = \frac{1}{1-\alpha} \ln(A_t) + \frac{\alpha}{1-\alpha} \ln\left(\frac{K_t}{Y_t}\right) + \ln\left(\frac{H_t}{L_t}\right)$$

- Let $y \equiv \frac{Y}{L}, k \equiv \frac{K}{Y}$, and $h \equiv \frac{H}{L}$ be the per capita expressions

- Differentiating with respect to t we get the following growth decomposition expression:

$$g_y = \frac{1}{1-\alpha} g_A + \frac{\alpha}{1-\alpha} g_k + g_h.$$

- For the US, we have the following growth accounting exercise from Jones (2016)

Table 3 Growth accounting for the United States

Period	Output per hour	Contributions from		
		K/Y	Labor composition	Labor-Aug. TFP
1948–2013	**2.5**	**0.1**	**0.3**	**2.0**
1948–1973	3.3	−0.2	0.3	3.2
1973–1990	1.6	0.5	0.3	0.8
1990–1995	1.6	0.2	0.7	0.7
1995–2000	3.0	0.3	0.3	2.3
2000–2007	2.7	0.2	0.3	2.2
2007–2013	1.7	0.1	0.5	1.1

Note: Average annual growth rates (in percent) for output per hour and its components for the private business sector, following Eq. (3).
Source: Authors calculations using Bureau of Labor Statistics, *Multifactor Productivity Trends*, August 21, 2014.

Development Accounting

- Cross-sectional analogue of Solow-style growth accounting

- Aim is to explain the huge income differences among countries

Some Observations

- The capital-output ratio does not vary too much across countries
  - Its average value is very close to one, and even the poorest country in the table, Malawi, is reported by the Penn World Tables to have a capital-output ratio very close to the US value.
  - So differences in physical capital contribute almost nothing to differences in GDP per worker across countries
- The contribution from educational attainment is larger, but still modest - e.g., countries like India and Malawi only see their incomes reduced by a factor of 2 due to differences in educational attainment (roughly, the poorest countries of the world have 4 or 5 years of education, while the richest have 13).

Table 6 Basic development accounting, 2010

	GDP per worker, y	Capital/GDP $(K/Y)^{\alpha/(1-\alpha)}$	Human capital, h	TFP	Share due to TFP
United States	1.000	1.000	1.000	1.000	–
Hong Kong	0.854	1.086	0.833	0.944	48.9%
Singapore	0.845	1.105	0.764	1.001	45.8%
France	0.790	1.184	0.840	0.795	55.6%
Germany	0.740	1.078	0.918	0.748	57.0%
United Kingdom	0.733	1.015	0.780	0.925	46.1%
Japan	0.683	1.218	0.903	0.620	63.9%
South Korea	0.598	1.146	0.925	0.564	65.3%
Argentina	0.376	1.109	0.779	0.435	66.5%
Mexico	0.338	0.931	0.760	0.477	59.7%
Botswana	0.236	1.034	0.786	0.291	73.7%
South Africa	0.225	0.877	0.731	0.351	64.6%
Brazil	0.183	1.084	0.676	0.250	74.5%
Thailand	0.154	1.125	0.667	0.206	78.5%
China	0.136	1.137	0.713	0.168	82.9%
Indonesia	0.096	1.014	0.575	0.165	77.9%
India	0.096	0.827	0.533	0.217	67.0%
Kenya	0.037	0.819	0.618	0.073	87.3%
Malawi	0.021	1.107	0.507	0.038	93.6%
Average	0.212	0.979	0.705	0.307	63.8%
1/Average	4.720	1.021	1.418	3.260	69.2%

The product of the three input columns equals GDP per worker. The penultimate row, "Average," shows the geometric average of each column across 128 countries. The "Share due to TFP" column is computed as described in the text. The 69.2% share in the last row is computed looking across the columns, ie, as approximately 3.5/(3.5 + 1.5).
Source: Computed using the Penn World Tables 8.0 for the year 2010 assuming a common value of $\alpha = 1/3$.

- The large contribution from TFP is verified by the last column of Table 6, where the share explained by TFP ranges from just under 50% for Singapore and Hong Kong to more than 90% for Malawi.

- To understand the "Share due to TFP" column, consider the last row of Table 6.

  - According to that row, the average country in the 128-country sample is just over 5 times poorer than the United States.

  - Essentially none of this difference (a factor of 1.02) is due to differences in K/Y, while a factor of 1.42 is due to differences in educational attainment, while 3.3 is due to TFP meaning that approximately $\frac{3.3}{3.3+1.44} \approx 69\%$

- For example, for Malawi, about a factor of 2 is due to inputs and a factor of 26 is due to TFP, meaning that $\frac{26}{26+2} \approx \mathbf{93\%}$ is due to TFP.

- This graph plots share of TFP in development accounting against GDP per worker

- We can see a clear pattern
  - In the poorest countries of the world, well over 80% of the difference in GDP per worker relative to the United States is due to TFP differences.
  - Moving across the graph to richer countries, one sees that less and less is due to TFP, and for the richest countries as a whole, TFP contributes around 50% of the differences.

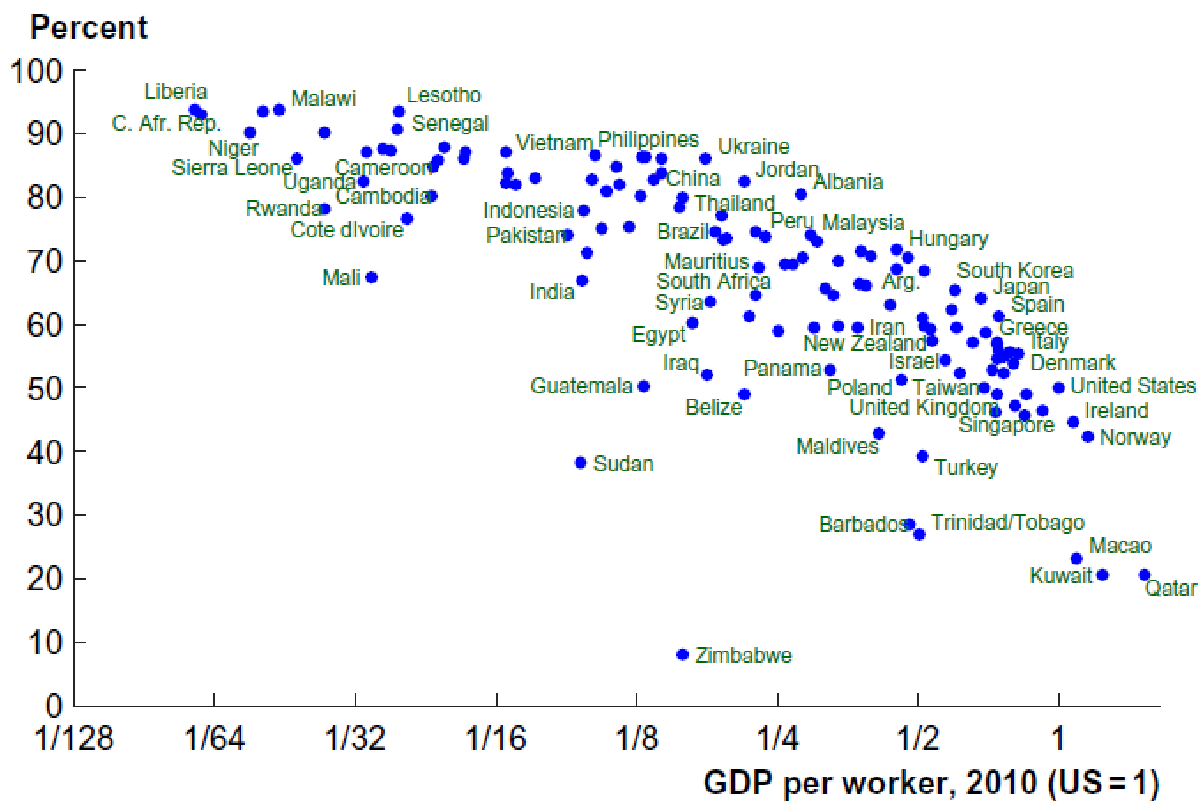

Fig. 30 The share of TFP in development accounting, 2010. Source: *Computed as described in the text and in Table 6 using the Penn World Tables 8.0 assuming a common value of* $\alpha = 1/3$.

- What could be the reason?
  - Richer countries are more similar and have converged to comparable per capita income levels
  - Less of variation to explain and so role of TFP not very high
  - Poorer countries are very dissimilar relative to richer countries and a lot of that is unexplained by measurable factors, so TFP plays a big role

Mankiw-Romer-Weil (1992), henceforth MRW, regression approach

- We base this on the discussion in Easterly-Levine (2001)

- Output is given by

$$Y_t = K_t^{\alpha} \left(A_t L_t \right)^{1-\alpha}.$$

- Let $L_t = L(1+n)^t$ where n is the exogenously given growth rate of population.

- Let $A_t = A(1+g)^t$ which is labour-augmenting technological change

- Let δ be the rate of depreciation

- Normalizing output and capital by the size of the effective labour force (AL) and denoting these by $\tilde{y} \equiv \frac{Y}{AL}$ and $\tilde{k} \equiv \frac{K}{AL}$ we get:

$$\tilde{y}_t = \tilde{k}_t^{\alpha}.$$

- From before, we have the capital accumulation equation:

$$\tilde{k}_{t+1} = \frac{s\tilde{k}_t^{\alpha} + (1-\delta)\,\tilde{k}_t}{1+n+g}.$$

- Therefore,

$$\triangle\tilde{k}_t = \tilde{k}_{t+1} - \tilde{k}_t = \frac{s\tilde{k}_t^{\alpha} - (n+\delta+g)\,\tilde{k}_t.}{1+n+g}$$

- Steady state:

$$\begin{aligned} s\left(\tilde{k}^*\right)^{\alpha} &= (n+\delta+\ g)\,\tilde{k}^* \\ \tilde{k}^* &= \left(\frac{s}{n+\delta+g}\right)^{\frac{1}{1-\alpha}}. \end{aligned}$$

- Steaty state per capita income

$$\tilde{y}^* = \left(\tilde{k}^*\right)^{\alpha} = \left(\frac{s}{n+\delta+\ g}\right)^{\frac{\alpha}{1-\alpha}}.$$

- Alternatively, as $\frac{Y}{AL} = \tilde{y}$, we can write

$$\frac{Y}{L} = A\left(\frac{s}{n+\delta+\ g}\right)^{\frac{\alpha}{1-\alpha}}$$

- Taking logarithms

$$\ln\left(\frac{Y}{L}\right) = \ln A + \frac{\alpha}{1-\alpha}\left[\ln s - \ln(n+\delta+\ g)\right].$$

- Let the second term be called MRW after Mankiw, Romer, and Weil (1992)

- We can allow different regions to have different intercepts (no country fixed effects)

- MRW coefficient $\frac{\alpha}{1-\alpha}$ is 0.44 which implies $\alpha = 0.31$ which is reasonable

- You can augment this model with having human capital as a separate input

- MRW showed that differences in saving rates, population growth rates, and rate of investment in human capital can explain nearly 80% variation in GDP level

- The main problem with this regression is it assumes that the distribution of A is independent of s, g etc across countries

- One can run panel data regressions (Islam, 1995)

- Obtains much higher rates of convergence

- The panel approach allows us to isolate the effect of "capital deepening" on the one hand and technological and institutional differences on the other, in the process of convergence.

- The results indicate that persistent differences in technology level and institutions are a significant factor in understanding cross-country economic growth.

- It becomes clear that if there had been no such differences, and countries differed only in terms of capital per capita, convergence would have proceeded at a faster rate

- Review by Caselli (2005) concludes that the answer to the development-accounting question - do observed differences in the factors employed in production explain most of the cross-country variation in income is: no.

- Hsieh and Klenow (2010) too note that there is a broad consensus that differences in human capital account for 10–30 percent of country income differences, physical capital accounts for 20 percent of country income differences, and residual TFP may be the biggest part of the story (accounting for 50–70 percent of country income differences).

- So the challenge is to understand productivity differences
  - Role of institutions and policies (Acemoglu, Johnson, Robinson, 2001)
  - Misallocation at the firm level can lower aggregate total factor productivity (Hsieh and Klenow, 2009)
  - Micro-level studies of specific markets (credit, land, labour, insurance)