NOTES ON SHARECROPPING

ETHAN LIGON

1. Introduction

We've been considering situations in which there's an incentive to share resources, but in which there's some impediment which makes it impossible to share everything both efficiently and equitably (the former in the sense of Pareto; the latter in the sense that all parties to the agreement must share voluntarily).

One of the most central sources of impediments to sharing found in all of economics is private information. The pre-eminent case of an institution in which private information plays a role is the case of the sharecropping contract. Dubois (2008) provides a brief survey of the recent literature. Here we'll pay particular attention to the principal-agent description of sharecropping arrangements.

2. The Principal-Agent Paradigm

A risk-neutral principal contracts with a risk-averse agent; the principal provides land, while the agent provides labor. Output is given by

$$y = g(x, a)e^{\epsilon}$$

where y is output, x is some non-labor investment (which can persist over time; Dubois interprets this as soil fertility), a is is the labor effort (action) taken by the agent, and ϵ is a shock, distributed so that $\mathrm{E}e^{\epsilon} = 1$.

The agent's utility depends on compensation c and labor effort a. These are usually assumed to be additively separable, with the agent's preferences represented by

$$U(c) - a.$$

It may appear to be very restrictive to assume that a only affects utility linearly, but since we haven't restricted the production function $g(x, a)$ there's no loss of generality in simply measuring labor effort in terms of utils.

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It's generally assumed that the agent's action a can't be observed by the principal, but that output can. Under these conditions of there being a "hidden action", it may then make sense for the principal to offer the agent a payment which depends only on observables such as output, and not on the unobservable action a.

3. Assuming a Linear Contract

Analysis of sharecropping arrangements often begins with an assumption that the agent's compensation is a linear function of output, with

$$c(y) = \alpha + \beta y.$$

This compensation rule pins down the behavior of the agent. He'll choose his action a to try to maximize his expected utility, or

$$\max_a \mathrm{E}U(\alpha + \beta y) - a$$

The solution to this problem gives effort as a function of the contract parameters, $a(\alpha, \beta)$. Then given this function, it's straightforward to state the principal's problem. She will maximize her expected profits by choosing the parameters α and β of the linear sharing contract subject to inducing the agent to agree to the contract *ex ante*, or

$$\max_{\alpha,\beta} (1 - \beta)\mathrm{E}[y|a(\alpha, \beta)] - \alpha$$

subject to

$$\mathrm{E}U(\alpha + \beta y) - a(\alpha, \beta) \geq \underline{U},$$

where $\underline{U}$ is the reservation utility of the agent. The solution to this problem generally hinges on choosing the share parameter β so as to get the trade-off between incentives and risk-sharing right, and then giving a lump-sum payment α just large enough to satisy the participation (or "individual rationality") constraint.

4. Endogenous Linearity

The restriction to a linear contract is sometimes justified by the claim that real-world sharecropping contracts are typically linear (though others would dispute this claim). However, another way to proceed is to take the position that if real-world contracts are in fact typically linear, then this is a fact which a model of sharecropping ought to explain, rather than an assumption to be imposed on the model.

In this spirit, we can tackle the problem of explaining linearity by starting with a more general model which *doesn't* assume linearity. Following Holmström (1979), we start by assuming a more general form for the random process which generates output, assuming that

y is governed by a cumulative distribution function $F(y|a, x)$, with a corresponding density $f(y|a, x)$. We assume that f is a continuously differentiable function of action a, and that it is everywhere positive.

As above, the agent's compensation is a function of observed output y, as the action a is private. But now we let the compensation rule be some general function $c(y)$. Now, given a compensation rule $c(y)$ we can write the agent's problem as

$$\max_a \int U(c(y))f(y|a, x)dy - a.$$

Provided that the agent's objective function is a concave function of a, then solution to the agent's problem of choosing an action will satisfy the first order condition

$$\int U(c(y))\frac{f_a}{f}(y|a, x)f(y|a, x)dy = 1, \tag{1}$$

where the factor $\frac{f_a}{f}(y|a, x)$ can be interpreted as the likelihood of y given a small increase in action a. Thus, the agent's first order condition balances the benefits (to him) of taking a higher effort against the cost of that higher effort.

Knowing that (provided that the concavity condition for the agent's objective is satisfied) the action taken by the agent will satisfy equation (1), the problem facing the principal can be formulated as

$$\max_{a,c(y)} \int [y - c(y)]f(y|a, x)dy$$

such that the agent's individual rationality constraint

$$\int U(c(y))f(y|a, x)dy - a \geq \underline{U}$$

is satisfied, and subject to (1) so that the agent takes the "recommended" action a.

Note that the compensation for the agent is chosen point-wise. The first order conditions for compensation satisfy

$$\frac{1}{U'(c(y))} = \lambda + \mu\frac{f_a}{f}(y|a, x).$$

Here μ is a multiplier on the "incentive compatibility" condition associated with the agent's choice of a. Thus, the second term of the right-hand side of this expression indicates exactly how the agent's compensation will depend on output, and governs his incentives, while the term λ, which is the multiplier on the individual rationality constraint, will be chosen so as to satify the agent's individual rationality constraint. Thus the multiplier μ is conceptually similar to the constant share parameter β which appeared in the linear problem, while the multiplier λ plays a role similar to the constant payment α.

Indeed, we might ask whether there are reasonable conditions on the utility function U and density f such that the optimal compensation scheme *is* in fact linear. Having posed the question, it's fairly clear how to go about answering it. It's not hard to see that for the compensation scheme to be linear, $U(c)$ must be an affine transformation of $\log(c)$. In fact, we can assume that it *is* $\log(c)$ without loss of generality here. Then we have

$$c(y) = \lambda + \mu \frac{f_a}{f}(y|a, x).$$

If $c(y)$ is in fact linear, then it follows that for some (α, β) we have

$$\alpha + \beta y = \lambda + \mu \frac{f_a}{f}(y|a, x),$$

but this can be rearranged so as to yield

$$(\alpha - \lambda + \beta y) f(y|a, x) = \mu f_a(y|a, x).$$

Now this is a partial differential equation in action a, which we can solve to compute the density f (remembering also that the density must integrate to one, and assuming that the support of y is the non-negative real line).

It's not difficult to verify that $f(y|a, x) = \left(\frac{y}{a}\right)^x \frac{1}{y\Gamma(x)} e^{-y/a}$ solves this pde, and so if the utility function is logarithmic and the density takes this form, then we obtain a linear contract as a *prediction* of the principal-agent model, with

$$\alpha = \lambda - \mu \frac{x}{a}, \qquad \beta = \frac{\mu}{a^2}.$$

Note that with this sharing rule the agent's problem is indeed a concave function of a, so that the approach of using the agent's first order conditions was valid.

Also note that the larger the investment x, the lower the lump-sum payment made to the agent (holding action fixed), while (again holding actions fixed), incentives are "higher-powered" when μ is large.

References

Dubois, P. (2008). sharecropping. In S. N. Durlauf and L. E. Blume. (Eds.), *The New Palgrave Dictionary of Economics*. Palgrave Macmillan.

Holmström, B. (1979). Moral hazard and observability. *Bell Journal of Economics 10*, 74–91.